

Statistical Properties of MPEG Video Traffic and their Impact on Bandwidth Allocation in Wireless ATM Networks

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Abstract - In this paper, we analyse the statistical data sets of several MPEG encoded videos and propose a model of the elements of a scene. The proposed model permits the characterisation of the elements of the stream scenes. The model can be used to allocate bandwidth dynamically on a scene basis and will result in a high capacity gain while guaranteeing the same and even better QoS. This model is particularly suited for MPEG encoded VBR traffic in wireless ATM networks.

Keywords: Wireless ATM, MPEG, Scene, QoS, CLR, VBR, statistical properties.

I. INTRODUCTION

In a wireless network, bandwidth is perhaps the most precious and limited resources of the whole communication system. Therefore, it is of extreme importance to use this resource in the most efficient way. Whenever a mobile terminal connects to a base station, the base station will allocate bandwidth to this mobile terminal. This bandwidth will remain constant throughout the duration of the connection.

The above approach is clearly inadequate for VBR sources. The bit rate of this kind of sources varies over time and they have most of the time a bursty nature. Compressed video sources are known to produce a Variable Bit Rate (VBR) with a high degree of burstiness, which needs specific resource management solutions, especially for guaranteed Quality of Service (QoS) networks.

If resource allocation is performed according to the peak cell rate of the VBR source, the network will be most of the time highly under-utilised when the peak-to-average rate ratios are high. The wireless bandwidth will be wasted and the wireless network will experience high call blocking and forced terminations. On the other hand, if resource allocation is performed based on the source mean cell rate, it is expected for the source to suffer from unacceptable losses and delays (especially for video sources imposing hard real-time constraints).

Several studies investigated the characterisation of MPEG traffic [8-10]. In [3], the authors concluded that the traffic probability density function (p.d.f.) of some VBR MPEG sources could be modelled by a Gamma distribution. In [4], the authors have shown that the Gamma and the Log-Normal distributions are good fits for the p.d.f. of I, P and B subsets of MPEG sequences. The Log-Normal distribution can also be used as a model for some VBR MPEG sources.

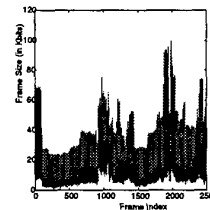


Fig. 1: Segment of the frame size sequence for Bond trace.

In the case that the model of the VBR source is known, we can calculate the required capacity¹ to have a certain CLR (Cell Loss Ratio). Even with this approach, big frames are more likely to be affected by a cell loss than small ones, which will affect the visual QoS. For example, consider the VBR source depicted in Fig. 1 offered to a bufferless switch (we consider only hard real-time services) on a wireless link of capacity C . Frames around 2000 will experience a very high cell loss which will be noticed by the user.

A way to solve such problem is the use of a dynamic bandwidth allocation algorithm. We propose to allocate bandwidth requirement for each scene depending on the GOP sizes mean and variance within the scene, where a scene represents a group of successive GOPs with close sizes. This will alleviate some of the problems described above. However, this can be done only if we have a characterisation of the elements of a scene. In this work, we will analyse the statistical data sets of several MPEG encoded videos and will propose a model of the elements of a scene. In this work, we consider the situation where a mobile terminal wants to send or receive an MPEG video stream over a wireless ATM link even if the ideas presented in this work are not specific to wireless ATM networks.

The paper is organised as follows. Section II introduces the scene concept. Section III presents a study of the distribution of the elements of a scene. In section IV, corresponding analysis and obtained results are presented. Section V presents evaluations of the Cell Loss Ratio. Conclusions and future directions are presented in section VI.

II. THE SCENE CONCEPT

An MPEG encoder generates three types of compressed frames: Intra-coded (I), Predictive (P), and Bi-directional (B)

¹ The terms 'capacity' and 'bandwidth' are used interchangeably throughout the paper.

frames. An *I* frame is encoded independently of other frames based on DCT (Discrete Cosine Transform) and entropy coding. A *P* frame uses a similar coding algorithm to *I* frames, but with the addition of motion compensation with respect to the previous *I* or *P* frame, and is used as a reference point for the next *P* frame. A *B* frame is an interpolated frame that requires both a past and a future reference frames (*I* or *P*). Typically, *I* frames require more bits than *P* frames. *B* frames have the lowest bandwidth requirement. After coding, the frames are arranged in a deterministic periodic sequence, for example “*IBBPBB*” or “*IBBPBBPBBPBB*”, which is called Group of Pictures (GOP).

From Fig. 1, it is observed that an MPEG trace consists of several segments such that the sizes of *I* frames in each segment are close in value. In [1,2], such segments were referred to as scenes. In this paper we consider scenes with respect to GOP sizes. The goal behind this choice is two folds. First, it permits to not distinguish between frame types (*I*, *P* or *B*). Second, it will allow for a uniform characterisation of the scene elements.

To model the length of a scene, the authors in [1,2] have proposed a method that computes scene duration using the fact that a “sufficient” difference between the sizes of two consecutive *I* frames is a strong indication of the start of a new scene. But this approach requires the availability of the VBR trace. It takes into account only *I* frames and do not permit a uniform characterisation of all frame types (*I*, *P* and *B*) within a scene.

In this work, we consider two requirements that will lead us to a new algorithm for determining the scene duration in an MPEG stream: First, the proposed algorithm must work “on-the-fly”, which means that the decision of determining the scene boundaries must only take into consideration the past GOPs. This will make our algorithm support MPEG streams independently from the knowledge of the trace. One advantage of such algorithm is the ability to handle MPEG streams for which we do not have a trace. The second requirement concerns the size of the first GOP in each scene, which has to be as close as possible to the mean GOP size of the scene. This will allow us to have a characterisation of the elements of a scene knowing only the size of the first GOP of that scene (as explained in section IV).

With respect to the above two requirement we compute scene duration differently (see Fig. 2). Let $\{GOP(j): j=1, 2, \dots\}$ be the GOP sequence in an MPEG stream. This sequence consists of the sizes of consecutive GOPs in a given MPEG trace. Suppose that the current scene is the i^{th} scene that started with the k^{th} GOP. The $(n+k+1)^{th}$ GOP of the sequence indicates the start of the $(i+1)^{th}$ scene if

$$|GOP(n+k+1) - GOP(k)| \geq T * GOP(k) \quad (1)$$

where T is a thresholds ($T \geq 0$). $n+1$ in this case represents the length of the i^{th} scene. Notice that the length of a scene is measured by the number of consecutive GOPs in that scene.

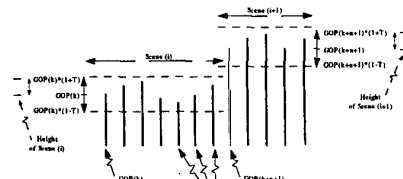


Fig. 2: Scene duration

With this definition of scene, all the GOP sizes within a scene i are located between $First_GOP(i)*(1-T)$ and $First_GOP(i)*(1+T)$. Where $First_GOP(i)$ is the size of the first GOP in scene i . Clearly, the value of T impacts the shape of the scene length distribution. It determines the amount of correlations between successive scenes; the larger these value, the less correlated the scenes. The value of the T parameter impacts also the number of scenes in a particular trace. Larger values of T produce smaller number of scenes. For example, for the MTV2 trace, a value of $T=20\%$ i.e. (0.2) produces 1200 scenes while a value of $T=80\%$ produces 100 scenes (see Fig. 3).

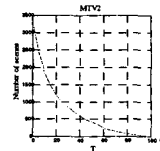


Fig. 3: Number of scenes for MTV2 trace

The traces used in our study were provided by Oliver Rose² [4]. Rose’s movies were taken from VCR tapes, and were digitised at rate of 25 frames/sec using a Sun Video card. The movies were compressed using MPEG-I [5,6] Berkeley’s software encoder [7]. Each MPEG video consists of 40.000 frames, which is equivalent to approximately half an hour.

III. GOP SIZE DISTRIBUTION WITHIN A SCENE

Based on our definition of a scene see (1), the sizes of GOPs within a scene could be considered close to each other. The GOP sizes fluctuate around an average value that represents the level of activity of the scene. Let $GOP(n)$ be the size of the n^{th} GOP in an MPEG stream. We suppose that $GOP(n)$ is the sum of two independent random variables:

$$GOP(n) = GOP^*(n) + GOPD(n) \quad (2)$$

Where $GOP^*(n)$ reflects the level of activity of the scene, while $GOPD(n)$ represents the fluctuation of the n^{th} GOP around $GOP^*(n)$. This assumption will simplify greatly the model.

The quantity $GOP^*(n)$ is constant for all GOPs in the same scene, and it varies from one scene to another. Hence, for the i^{th} scene that started with the k^{th} GOP, we have

$$GOP^*(k) = GOP^*(k+1) = \dots = GOP^*(k + N_i - 1) = GOP_S^*(i) \quad (3)$$

N_i represents the length of the i^{th} scene. Notice that the length of a scene is measured by the number of consecutive

² The traces can be obtained from the ftp site <ftp://ftp-info3.informatik.uni-wuerzburg.de> in the directory /pub/MPEG/.

GOPs in that scene.

Suppose that $\{GOP_S(i) : i=1,2,\dots\}$ is a sequence of i.i.d. random variables with common *pdf* f . The histogram of $\{GOP_S(i)\}$ for the MTV1 trace is shown in Fig. 4. The shape of this histogram suggests the use of a Log-Normal or a Gamma fit. Solid line represents the Log-Normal fit for the empirical data, dash-dot line represents the Gamma fit.

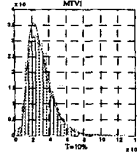


Fig. 4: The histogram of $\{GOP_S(i)\}$ for MTV1 trace with $T=10\%$

Consider $GOPD(n)$ the quantity representing the variation of the size of the n^{th} GOP around $GOP^*(n)$. For a given trace, we compute the empirical sequence $\{GOPD(n)\}$, where $GOPD(n) = GOP(n) - GOP_S(i_n)$ (i_n is the scene index to which the n^{th} GOP belongs). Note that $\{GOPD(n)\}$ was assumed as independent of $\{GOP_S(i)\}$, and thus invariant with respect to scene changes, and depends only on the T parameter.

The histogram of $\{GOPD(n)\}$ for the MTV1 trace is shown in Fig. 5. The shape of this histogram suggests the use of a Normal fit. Therefore, we set $f_{GOPD} = \mathcal{N}(0, \sigma_{GOPD}^2)$. Since $\{GOPD(n)\}$ is the variation of the GOP sizes around the mean size value of the scene to which each GOP belongs, the mean value of $\{GOPD(n)\}$ is equal to zero. Thus the GOP sizes within a scene i can be modelled by a normal distribution with mean $\mu = GOP_S(i)$ and variance $\sigma^2 = \sigma_{GOPD}^2$ ($\mathcal{N}(\mu, \sigma^2)$). μ varies from one scene to another (since it reflects the level of activity of the scene) while σ is invariant to scene changes and depends only on the T parameter.

IV. THE INFLUENCE OF THE T PARAMETER

Our ultimate goal is to allocate bandwidth dynamically on a per scene basis knowing the first GOP of the scene only. For that purpose, it is important to characterise the elements of the scene. As this characterisation depends on the T parameter, the study of the influence of the T parameter is of great importance.

Let us recall that T is the parameter that determines the height of the scenes (see Fig. 2). The value of T impacts the shape of the scene length distribution. It determines the amount of correlations between successive scenes; the larger these value, the less correlated the scenes. The value of the T parameter impacts also the number of scenes in a particular trace. Larger values of T produce smaller number of scenes (see Fig. 3).

Our goal is the characterisation of the GOPs within a scene. The idea is to see if from the study of MPEG traces we can approximate or even calculate the mean and the variance of the GOPs within a scene knowing only the size of the first GOP of that scene. This will allow a characterisation of the entire scene based on the size of the first GOP, which can be used to allocate bandwidth for the scene.

We compute the value $GOP(n) - GOP^*(n)$ for every n as the difference between each GOP size and the mean size of the scene to which the GOP belongs. Let us recall that

$$GOPD(n) = GOP(n) - GOP^*(n) \text{ for all } n.$$

$GOPD$ depends on the value of the T parameter. For values of T below 200%, the shape of $\{GOPD\}$ suggests the use of a normal fit. Values above 200% are not important for us since the entire MPEG stream will not have many scenes, and hence, will not profit from the scene-based bandwidth allocation. Indeed, for T above 200%, the majority of studied streams are composed of a single scene.

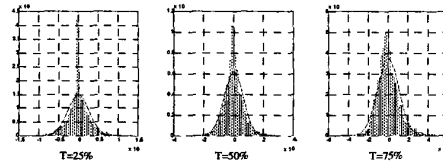


Fig. 5: GOPD for $T=25\%$, 50% and 75% for MTV1 trace

Fig. 5 depicts the histogram for empirical sequences $\{GOPD(n)\}$ for different values of T, namely 25%, 50% and 75% for the MTV1 trace. Solid lines represent the Normal fits for the empirical data.

The normal distribution f_{GOPD} is fully characterised by the mean μ_{GOPD} and the variance $(\sigma_{GOPD})^2$ of the empirical sequence $\{GOPD\}$. Since $\{GOPD(n)\}$ depends on the T parameter, the values μ_{GOPD} and σ_{GOPD} depend also on the T parameter.

We compute μ_{GOPD} and σ_{GOPD} for different values of T (from $T=0\%$ to 200% step 1%). The results are presented Fig. 6. Since $GOP^*(n)$ represents the mean GOP size of the scene to which $GOP(n)$ belongs, μ_{GOPD} is equal to zero for all T.

Fig. 6 shows the variation of σ_{GOPD} as a function of T for the ATP, ASTERIX and MTV1 traces.

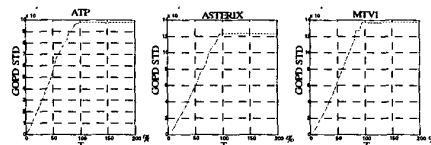


Fig. 6: GOPD STD for ATP, ASTERIX and MTV1 traces

When T is near 0 ($<10\%$), the scenes contains only few GOPs (we suppose that the GOP sizes are not constant since we consider VBR traffic), σ_{GOPD} is also near 0. σ_{GOPD} will increase with the value of T until some maximum value and then it remains constant (see Fig. 6). The constant value, σ_{GOP} , is the standard deviation (STD) of the entire stream. Indeed, when T is very high the entire stream is considered as one scene and the STD of $\{GOPD\}$ is nothing but σ_{GOP} . Since there is some value T_M from which there is only one scene in the entire stream, we can set $\sigma_{GOPD} = \sigma_{GOP}$ for all $T \geq T_M$.

It is worth noting that the value of σ_{GOPD} increases linearly from zero to σ_{GOP} . Also, σ_{GOPD} begins to stabilise, sometimes, before T reaches T_M . This occurs for values of T for which the entire stream contains only few GOPs (2 or 3). Let the

first value of T for which this stabilisation occurs be T_m .

We can then model the value of σ_{GOPD} by

$$\sigma_{GOPD} = T * \sigma_{GOP} / T_m \text{ for } T \leq T_m \text{ and } \sigma_{GOPD} = \sigma_{GOP} \text{ for } T \geq T_m \quad (4)$$

From the study of MPEG traces, we notice that the value of T_m is always around 90%. If the maximum GOP size, GOP_{MAX} , and the minimum GOP size, GOP_{min} , of the entire GOP stream are known, we can calculate the value of T_M as follows:

$$T_M = \text{MAX}[(1 - GOP_{min}/GOP(1)), (GOP_{MAX}/GOP(1) - 1)] \quad (5)$$

Where $\text{MAX}[X, Y]$ is the maximum operator, and $GOP(1)$ is the size of the first GOP of the stream.

Table 1 shows the values of T_M and T_m for different MPEG streams. Note that values of T_m are only approximations found while plotting the variation of σ_{GOPD} as a function of T. Table 1 shows also the number of scenes for the MPEG traces with $T = T_m$.

We consider the variance and the average of the films as known values for two reasons: Either we have these films beforehand, hence we can calculate these values, or use an encoder that allows us to specify a desired variance. A number of works [12,16] have been done on designing rate control (or rate shaping) mechanisms to enforce the encoder to respect some predefined characteristics like a certain mean rate and target variance.

Our ultimate goal is to allocate bandwidth dynamically on a per scene basis knowing the first GOP of the scene only. For that purpose, it is important to characterise the elements of the scene. As shown earlier, the GOP sizes within a scene can be modelled by a normal distribution. We must find μ^* and σ^* that approximate or overestimate μ and σ for each scene. We can use these values to allocate bandwidth for the scene, and if $\mu^* \geq \mu$ and $\sigma^* \geq \sigma$ for each scene then user requirements will not be violated.

Table 1 : T_M and T_m values for some traces

Film	T_M	T_m	Number of scenes for $T = T_m$
News1	84.90%	84.90%	1
MrBean	88.49%	88.49%	1
Lambs	89.76%	89.76%	1
MTV1	130.13%	94%	2
Simpsons	158.90%	97%	2
ATP	160.20%	97%	2
Soccer	160.37%	91%	2

For MPEG streams, for which we have already traces, μ is accurately computed based on those traces. However, for MPEG streams for which we don't have traces beforehand μ must be approximated.

The STD σ can be calculated using (4). To check if we can replace the mean GOP size with the size of the first GOP without violating user requirements (a specified CLR), we compute the difference $\{GOPDS\}$. This corresponds to check if $\mu^*(s) = \text{First_GOP}(s)$ is viable. We compute the percentage that represents $GOPDS(s) = (\mu^*(s) - \mu(s))$ to the height of scene s . The height of a scene s is defined by $\text{First_GOP}(s)*T$

(see Fig. 2). This percentage is determined as follows:

$$\text{PERC}(s) = [\text{GOPDS}(s) / (\text{First_GOP}(s)*T)]$$

As shown in Fig. 7 PERC is not always positive. Similar results were found for all values of T below 100% and for all other MPEG traces. This means that for many scenes the size of the first GOP of the scene is smaller than the mean GOP size within the scene. Thus if $\mu^*(s) = \text{First_GOP}(s)$ is used to evaluate the bandwidth required to satisfy a certain CLR, user requirements will not be guaranteed.

This problem can be solved by correcting the value of μ^* . We propose to add the STD σ_{GOP} of the entire stream, obtaining consequently $\mu^*(s) = \text{First_GOP}(s) + \sigma_{GOP}$ for each scene s .

As all GOP sizes within a scene s are below $\text{First_GOP}(s)*(1+T)$, we set $\mu^*(s) = \text{MIN} [\text{First_GOP}(s) + \sigma_{GOP}, \text{First_GOP}(s)*(1+T)]$ for each scene s . $\text{MIN}[X, Y]$ here represents the minimum operator.

The new percentage that represents $(\mu^*(s) - \mu(s))$ to the height of scene s is now computed. Fig. 8 shows that this new percentage is always positive. Similar results are obtained for all values of T below 100% and for all other studied MPEG traces.

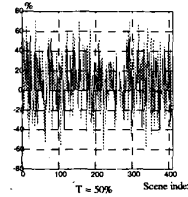


Fig. 7: PERC with $T = 50\%$ and $\mu^*(s) = \text{First_GOP}(s)$ for the MTV1 trace

The new value of μ^* can be used to characterise the elements of each scene without violating user requirements (as explained in section V).

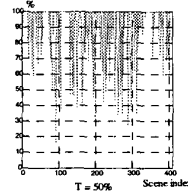


Fig. 8: PERC with the new value of μ^* for the MTV1 trace

In [11], we have proposed a dynamic bandwidth allocation algorithm allowing for a capacity gain of 82.06% for $T=10\%$ and a required $\text{CLR}=10^{-10}$. This means that if the static allocation approach (based on the characterisation of the entire MPEG stream) uses a particular amount B of bandwidth to have a $\text{CLR}=10^{-10}$, our algorithm uses only 17.94 % of B to satisfy the same CLR requirement.

The proposed algorithm allocates much less capacity than the constant allocation scheme while guarantying the required QoS. This allows distributing the cell loss over all the film scenes while with constant allocation methods the cell loss is concentrated within big frames. Consequently, big GOPs are not disadvantaged compared to small ones, and hence the

visual quality of the film is improved. It is worth noting that in this work the problem of call admission is not addressed and that the capacity gain is obtained supposing that the required capacity is always available.

V. CELL LOSS RATIO EVALUATION

Consider a hard real-time VBR service where the stream produced by the VBR source is directed to a bufferless switch on a wireless link of capacity C . Let this VBR source bit-rate at time t be R_t . The Cell Loss Ratio (CLR) can be estimated by the fluid approximation [12] as follows:

$$CLR = \frac{E\{R_t - C\}^+}{E\{R_t\}} \quad (6)$$

Where $E\{.\}$ represents the expectation operator and X^+ is defined as $X^+ = X$ if $X > 0$ and $X^+ = 0$ if $X < 0$. If the p.d.f. of the source rate is defined by $f(u)$, then (6) can be written as:

$$CLR = \frac{\int_0^{\infty} (u - C) f(u) du}{\int_0^{\infty} u f(u) du} \quad (7)$$

For a normal distribution $\mathcal{N}(\mu, \sigma)$ with a p.d.f. f and a cumulative distribution function (c.d.f.) F and using (7) we obtain:

$$CLR = \frac{\mu^*(1 - F(C)) + \sigma^2 * f(C) - C*(1 - F(C))}{\mu} \quad (8)$$

Using (8) we can calculate the required capacity C for a pre-specified CLR. The performed simulations show that the required CLR is always respected. We have allocated bandwidth to each scene s supposing a normal distribution of the GOPs within the scene as stated in section III. We have taken $\mu^*(s) = First_GOP(s) + \sigma_{GOP}$ and σ^* as in (4).

Fig. 9 shows the obtained CLR for the ATP trace and for different required CLR's. We remark that the required CLR is always respected. For values of CLR below 10^{-4} , the obtained CLR is always below 10^{-10} . Similar results are obtained for all the studied MPEG traces. We have computed the obtained CLR for 16 MPEG streams for different values of T (from $T=10\%$ to 100% step 10%) and different values of CLR (from 10^{-10} to 0.1 step 0.1). We always have the same and even better CLR (for a CLR below 10^{-3}).

VI. CONCLUSION

In this paper, we analysed the statistical data sets of several MPEG encoded videos and proposed a model of the elements of the stream scenes. The proposed model permits the characterisation of the elements of the stream scenes, which can be used to allocate bandwidth dynamically for each scene and will result in a high capacity gain while guaranteeing the same and even better QoS. Taking $\mu^*(s) = First_GOP(s) + \sigma_{GOP}$ as an approximation of μ for each scene s is a good evaluation of the mean GOP size within the scene s . Indeed, this results in a lower CLR. However, this approximation is not the best evaluation because it overestimates the mean μ .

Future work will involve studying a better estimation of μ , which, while guaranteeing the CLR requirement, will further increase the capacity gain by allocating less bandwidth. Such

study is particularly important for MPEG streams for which we don't have traces beforehand. For MPEG streams, for which we have already traces, μ is accurately computed based on those traces. The impact of a connection admission control mechanism on the obtained results is also of great importance.

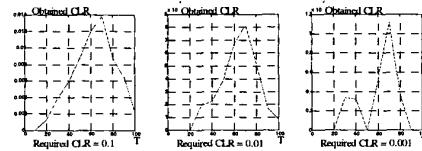


Fig. 9: obtained CLR for ATP trace

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