DATA SERVICE PERFORMANCE ANALYSIS IN GPRS SYSTEMS

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Abstract - In this paper we describe an analytical approach for deriving the packet delay distribution in a cell of a wireless network operating based on the general packet radio service (GPRS) standard. Based on that, the average packet delay and packet loss probability are also computed. Our approach is based on a decomposition of system behavior into short-term and long-term behaviors to simplify the analytical modeling. In addition to the effect of voice call handoffs, the impact of packet forwarding and dedicated data channels on data service performance is also taken into consideration. The performance estimates produced by the analytical approach are compared with those generated by simulation experiments. The comparison results confirm the relative accuracy of the analytical approach.

Keywords - General packet radio service (GPRS), data service, performance evaluation, delay distribution.

I. INTRODUCTION

General Packet Radio Service (GPRS) [1] is a new bearer service designed as an extension to the GSM network, which greatly improves and simplifies the wireless access to packet data networks such as the Internet [2]. In this paper, we develop an analytical approach to compute the packet delay and loss probability of data service in a GSM/GPRS cellular network.

In a more general context, voice and data integration in wireless networks has been recently investigated by wireless research community. An integrated voice/data wireless system with finite buffer for data traffic has been considered in [3], where the system is described by a two-dimensional Markov chain. Then, balance equations are given which can be numerically solved to find the interesting performance parameters. A similar system based on the movable boundary approach is investigated in [4]. Handoff effect of voice traffic is considered but it is assumed that data packets waiting in queue will not handoff. Fluid flow analysis has been applied in [5] while only the effect of voice handoff has been taken into consideration. More specifically, a GPRS system with voice handoffs is investigated in [6] and [2]. A two-dimensional continues time Markov chain has been formulated in [6] to describe the system behavior. Queueing of voice calls has been investigated in detail in [2] without considering the queueing of data traffic.

This paper focuses on the performance of GPRS data service with buffering and dedicated channels for data traffic while considering the effect of reneging data packets due to the corresponding user handoff. The key feature of our approach is that it considers the impacts of queueing, reneging and dedicated channels on data service performance while provides product-form solutions for major performance parameters. Our objective is to show the effect of handoffs on the performance of GPRS data traffic which is neglected by other researchers (please refer to [7] for an extended version of this paper).

The rest of the paper is organized as follows. In section II, we describe the system model and clarify our assumptions. System analysis is presented in section III. Section IV presents some numerical results, and finally, section V concludes the paper.

II. SYSTEM MODEL

The system under consideration is a GSM/GPRS network, in which the users move along an arbitrary topology of M cells according to the routing probability r_{ij} (from cell i to cell j). In each cell k with c_k channels, d_k channels are reserved for GPRS data traffic and the rest, $c_k - d_k$ channels, are shared by voice and data traffic. In the shared part, voice traffic has priority over data traffic and can preempt data traffic. The assumptions and parameters involved in this model are stated below.

1) The new voice call and data packet arrivals into cell k are Poisson distributed with rates $\lambda_n^v(k)$ and $\lambda_n^d(k)$.
2) The residence time of a mobile station in cell k is assumed to be exponentially distributed with means $1/\eta_k^v$ and $1/\eta_k^d$ for voice and data, respectively. A queued data packet is removed from the queue and is forwarded to the new cell, if the corresponding mobile station leaves the service area of the original cell before transmission. We assume that packet forwarding can not happen during the actual packet transmission due to the short length of data packets. Throughout this paper we use terms packet forwarding and packet handoff interchangeably.
3) The handoff call arrivals into cell k are assumed to be Poisson distributed with rates $\lambda_h^v(k)$ and $\lambda_h^d(k)$.
4) The transmission time of a GPRS packet in cell k is assumed to be exponentially distributed with mean $1/\mu_k^d$, where one channel serves the packet. The call holding time of a voice call is assumed to be exponentially distributed with mean $1/\mu_k^v$.
5) We define *mobility factor* to be the ratio of the mean service (or call holding) time to the mean residence

time, i.e., $\alpha_k^v = \eta_k^v/\mu_k^v$ and $\alpha_k^d = \eta_k^d/\mu_k^d$.

6) A finite buffer with capacity b_k packets is provided in each cell k for GPRS packets only.

In the real word, the cell residence time distribution may not be exponential but exponential distributions are widely used in research papers [2–6] and do provide the mean value analysis, which indicates the performance trend of the system. Our focus in this paper is on the performance of GPRS data traffic, hence handoff prioritization is not considered for voice calls.

III. Analysis

Performance analysis of the GPRS can be accomplished by describing the system as a two-dimensional Markov chain corresponding to voice and data dynamics. This approach is computationally too complex and usually there is no closed-form expression for the performance parameters [3, 4, 6]. On the other hand, performance analysis based on simulations is prohibitively time consuming due to the significant difference in the time-scale of voice and data dynamics.

In GSM/GPRS systems the mean holding time of voice calls is much larger than the mean service time of data packets, hence voice calls evolve slowly compare to the data buffer dynamics. Therefore, the data queueing process exhibits transient behavior immediately after any change in the number of active voice calls, but will eventually settle into steady-state behavior. As an alternative approach, we take the advantage of this quasi-stationary behavior of the data queueing process to approximately evaluate the performance of data service in GPRS systems by decomposing the system behavior into short-term and long-term behaviors.

A. Long Term Behavior

Let $\boldsymbol{p}_k$ denote the stationary state probability vector of the Markov chain describes the number of active voice calls in cell k. Using balance equations, it is obtained that

$$\boldsymbol{p}_k(i) = \frac{1}{i!}\left(\frac{\lambda_n^v(k)+\lambda_h^v(k)}{\mu_k^v+\eta_k^v}\right)^i \boldsymbol{p}_k(0), \qquad 1 \le i \le c_k - d_k \tag{1}$$

where $\boldsymbol{p}_k(0)$ can be found using the normalizing condition $\sum_{i=0}^{c_k-d_k} \boldsymbol{p}_k(i) = 1$. Consequently, the call blocking probability of voice calls is given by

$$B_k = \boldsymbol{p}_k(c_k - d_k). \tag{2}$$

In a GPRS system, the number of channels that can be assigned to serve data packets is the sum of dedicated data channels, d_k, and those channels not used by voice calls. Let $\boldsymbol{\pi}_k$ denote the steady state probability vector of the number of channels that are available for GPRS data service in cell k, then

$$\boldsymbol{\pi}_k(m) = \boldsymbol{p}_k(c_k - d_k), \qquad m = d_k, d_k+1, \ldots, c_k. \tag{3}$$

B. Short Term Behavior

Assume that the number of channels serving GPRS packets in a cell k is fixed and equal to m with probability $\boldsymbol{\pi}_k(m)$ given by (3). Each packet requires one channel for transmission. Let s_i denote the state of the cell where $i\,(0 \le i \le m + b_k)$ indicates the number of GPRS packets in the cell which are being served or waiting in the queue. Also, let $\delta_k^m(i)$ denote the transition rate from state s_i to state s_{i-1} when m channels serve the data packets, i.e.,

$$\delta_k^m(i) = \begin{cases} i\mu_k^d, & 0 \le i \le m \\ m\mu_k^d + (m-i)\eta_k^d, & m \le i \le m + b_k \end{cases} \tag{4}$$

Using balance equations, the steady-state probability vector $\boldsymbol{q}_k^m$ is given by

$$\boldsymbol{q}_k^m(i) = \prod_{j=1}^{i}\left(\frac{\lambda_n^d(k)+\lambda_h^d(k)}{\delta_k^m(j)}\right)\boldsymbol{q}_k^m(0), \quad 1 \le i \le m + b_k \tag{5}$$

where $\boldsymbol{q}_k^m(0)$ can be found using the normalizing condition $\sum_{i=0}^{m+b_k} \boldsymbol{q}_k^m(i) = 1$. A packet is lost when the data buffer is full upon its arrival. Therefore, the packet loss probability, L_k^m, is simply given by

$$L_k^m = \boldsymbol{q}_k^m(m + b_k). \tag{6}$$

C. Packet Delay Distribution

Define the packet delay as the time between the acceptance of a packet in a cell and the time its service starts in that cell. Let W_k^m denote the delay of an arriving packet in steady-state where m channels are serving GPRS packets in cell k. We are interested in finding the probability distribution of W_k^m

$$F_{W_k^m}(\tau) = \Pr(W_k^m \le \tau) \tag{7}$$

or, equivalently, the probability density function

$$f_{W_k^m}(\tau) = \frac{d}{d\tau}F_{W_k^m}(\tau). \tag{8}$$

where $\tau \in \Re^+$ throughout this paper.

Consider the service part of the cell. Let S denote the duration between the time that all m data channels become busy and the time that the first channel is released. The probability distribution function of S is expressed as

$$F_S(\tau) = 1 - e^{-m\mu_k^d\tau} \tag{9}$$

The cell residency time of a packet, R, is determined by the time the corresponding portable remains in the cell. Therefore, the probability distribution function of R is expressed as

$$F_R(\tau) = 1 - e^{-\eta_k^d\tau} \tag{10}$$

Assume that a data packet d_i arrives to the cell when the cell is in state s_i. If $0 \le i < m + b_k$ then packet d_i is accepted

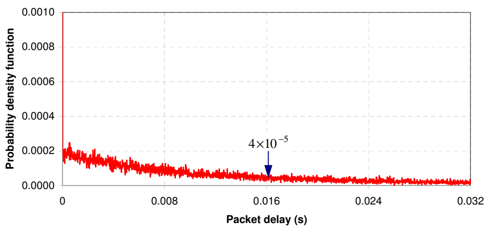


Fig. 1. Packet delay density function ($f_{W_k}(\tau)$).

and the cell state will change into s_{i+1}. If $i < m$ then d_i will be immediately served, otherwise it must wait in the queue for $(i-m+1)$ packet departures (either transmission or handoff). Let W_i denote the delay of d_i under the condition that d_i will not handoff prior to its service commences.

Suppose we temporarily view the cell as consisting of i packets (which are ahead of d_i). Then the time required for the packet population to decrease from j to $j-1$ is exponentially distributed with rate parameter $\delta_k^m(j)$. The probability that the arriving packet is not forwarded to another cell during the interval of time required to drive the packet population from j to $j-1$ is given by $\delta_k^m(j)/\delta_k^m(j+1)$ for $m \leq j \leq i$. Therefore, the probability that d_i does not handoff into another cell before it is being transmitted, β_i, is given by

$$\beta_i = \Pr(W_i \leq R) = \prod_{j=m}^{i} \frac{\delta_k^m(j)}{\delta_k^m(j+1)} = \frac{\delta_k^m(m)}{\delta_k^m(i+1)} \quad (11)$$

where $\beta_i = 1$ for $i < m$.

Clearly $W_i = 0$ if $i < m$, thus for the rest of the derivation, we consider only the case of $m \leq i < m + b_k$ to simplify the equations. According to Movaghar [8] we can write

$$\Pr(W_i \leq \tau) = \Pr(W_{i-1} + S \leq \tau \,|\, W_{i-1} \leq R) \quad (12)$$

where S and R are defined by (9) and (10), respectively. Define $F_{W_i}(\tau)$ and $f_{W_i}(\tau)$ similar to (7) and (8). Notice that $f_{W_i}(\tau) = 0$ for $i < m$. Using (12) we have

$$F_{W_i}(\tau) = \frac{\int_0^\tau F_S(\tau - t)(1 - F_R(t)) f_{W_{i-1}}(t)\, dt}{\Pr(W_{i-1} \leq R)}. \quad (13)$$

After some algebraic manipulations (please refer to [7] for details) it is obtained that

$$f_{W_i}(\tau) = \frac{\prod_{j=0}^{i-m} \delta_k^m(m+j)}{(i-m)!} \left(\frac{1 - e^{-\eta_k^d \tau}}{\eta_k^d} \right)^{i-m} e^{-m\mu_k^d \tau}. \quad (14)$$

Finally, the steady-state probability density of W_k^m is given by

$$f_{W_k^m}(\tau) = \sum_{i=m}^{m+b_k-1} \boldsymbol{q}_k^m(i) f_{W_i}(\tau). \quad (15)$$

D. Average Packet Delay

As expressed in (14), packet delay is exponentially deceasing with time, hence, an average value may be a sufficiently accurate measure of the actual delay. Fig. 1 shows the actual measurements from a simulated GPRS network in which $c_k = 7$, $d_k = 1$, $b_k = 20$ and $\alpha_k^v = 1.8$ (please see section IV for detailed simulation parameters). For this simulation, the average packet delay found to be $16\,ms$ where the offered GPRS load and the average packet transmission time were set to 2 Erlangs and $18\,ms$, respectively. As shown in Fig. 1, the probability of having packet delays greater than the average delay is very small. However, our approach is approximate and such estimations will not affect its performance severely.

Consider packet d_i from previous subsection. We are interested in finding the average delay of d_i denoted by $E[W_i]$ under the condition that d_i will not hand off before transmission. Let t_j denote the time required for the packet population to decrease from j to $j-1$ ($m \leq j < m + b_k$). Hence, the delay of d_i is $T_i = \sum_{j=m}^{i} t_j$, where t_j is exponentially distributed with rate $\delta_k^m(j)$, therefore $E[t_j] = 1/\delta_k^m(j)$. Consequently, we have

$$E[W_i] = E[T_i | W_i \leq R] = \frac{E[T_i \text{ and } W_i \leq R]}{\Pr(W_i \leq R)} \quad (16)$$

where

$$E[T_i \text{ and } W_i \leq R] = \left(\frac{\delta_k^m(m)}{\delta_k^m(i+1)} \right) \sum_{j=m}^{i} 1/\delta_k^m(j). \quad (17)$$

Substituting (17) in (16) gives $E[W_i] = \sum_{j=m}^{i} 1/\delta_k^m(j)$. Finally, the steady-state average packet delay $E[W_k^m]$ is expressed as

$$E[W_k^m] = \sum_{i=m}^{m+b_k-1} \boldsymbol{q}_k^m(i) \sum_{j=m}^{i} 1/\delta_k^m(j) \quad (18)$$

E. Handoff Arrival Rate

We use an iterative algorithm to find the voice call and data packet handoff arrival rates into each cell k as follows.

1) Voice Call Handoff Rate: In cell j, the probability that a voice call will attempt to hand off is $P_H^v(j) = \eta_j^v/(\mu_j^v + \eta_j^v)$, hence the handoff rate of voice calls out of cell j is given by

$$(\lambda_v^j + \nu_v^j)(1 - B_j) P_H^v(j). \quad (19)$$

Using the routing probabilities r_{jk}, the handoff rate of voice calls into cell k is given by

$$\lambda_h^v(k) = \sum_{j \neq k} r_{jk} \left[\lambda_n^v(j) + \lambda_h^v(j) \right] \left[1 - B_j \right] P_H^v(j). \quad (20)$$

Starting with an initial value of $\lambda_h^v(k)$, iteration can be used to obtain the steady-state value of $\lambda_h^v(k)$.

2) Data Packet Handoff Rate: Following the same approach for the handoff voice calls, the handoff rate of data packets moving into cell k can thus be derived as

$$\lambda_h^d(k) = \sum_{j \neq k} r_{jk} \left[\lambda_n^d(k)(j) + \lambda_h^d(k)(j) \right] P_H^d(j) \quad (21)$$

where $P_H^d(j)$ is the probability that a waiting data packet in a cell will attempt to hand off and is given by $P_H^d(j) = \sum_{i=m}^{m+b_j-1} (1 - \beta_i) \boldsymbol{q}_j^m(i)$.

F. Approximate System Behavior

The approximate system behavior is obtained by the aggregation of short term behavior with respect to the long term behavior. Let f_{W_k}, $E[W_k]$ and L_k denote the packet delay density function, average packet delay and loss probability, respectively, in cell k. We may write

$$E[W_k] = \sum_{m=d_k}^{c_k} \boldsymbol{\pi}_k(m) E[W_k^m] \quad (22)$$

$$L_k = \sum_{m=d_k}^{c_k} \boldsymbol{\pi}_k(m) L_k^m \quad (23)$$

$$f_{W_k}(\tau) = \sum_{m=d_k}^{c_k} \boldsymbol{\pi}_k(m) f_{W_k^m}(\tau) \quad (24)$$

where $f_{W_k^m}$, L_k^m and $E[W_k^m]$ are given by (15), (6) and (18).

IV. Numerical Results

An event-driven simulation was developed to verify the accuracy of this analysis. The simulation considered a two-dimensional GSM/GPRS network, in which the coverage area is partitioned into seven cells. Opposite sides wrap-around to eliminate the finite size effect. We assumed that mobile users move along the cell areas according to a uniform routing table, i.e. all cells are equally chosen for handoff, although the simulation can accommodate general cases. To simplify our results, only the exponential cell residency and transmission times were considered. Besides, for ease of illustrating the results, we assumed that for any cell k

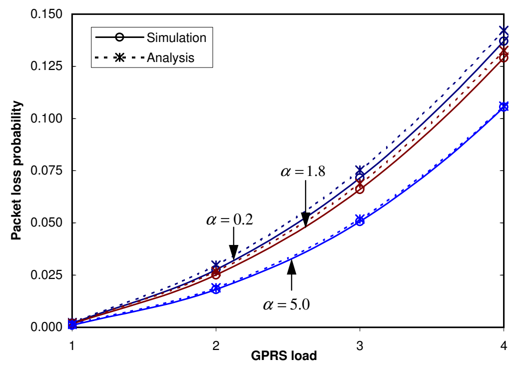
$$\rho_v = \frac{\lambda_n^v(k)}{\mu_k^v} = \frac{\lambda_v}{\mu_v}, \quad \rho_d = \frac{\lambda_n^d(k)}{\mu_k^d} = \frac{\lambda_d}{\mu_d} \quad (25)$$
$$c = c_k, \quad d = d_k, \quad b = b_k, \quad \alpha = \alpha_k^v.$$

where $\rho_v(\rho_d)$ indicates the offered voice (data) load. In this case, all the cells in the network exhibit the same performance. We assumed that there is one frequency carrier (or seven channels) per cell, i.e., $c = 7$. Furthermore, in all the cases that have been simulated, $\rho_v = 3$, $1/\mu_v = 180s$ and $\mu_d/\mu_v = 10^4$. This set of parameters ensures an acceptable level of call blocking for voice calls ($\approx 5\%$). Table 1 shows the simulated configurations and their corresponding voice call blocking probability computed through analysis and simulation (with 95% confidence intervals).

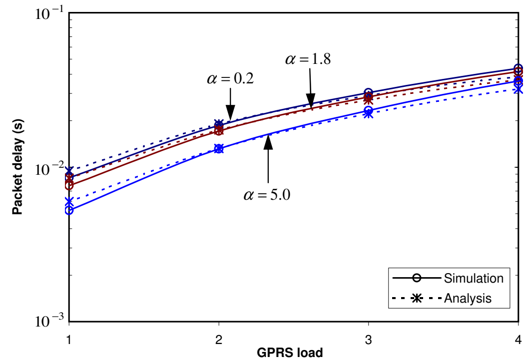
Table 1
Voice call blocking probability.

Profile		P_B	
Effect	Parameter	Simulation	Analysis
Mobility ($b = 20, d = 1$)	$\alpha = 0.2$	0.045 ± 0.005	0.051
	$\alpha = 1.8$	0.041 ± 0.004	0.041
	$\alpha = 5.0$	0.031 ± 0.002	0.032
Buffer size ($\alpha = 1.8, d = 1$)	$b = 20$	0.041 ± 0.004	0.041
	$b = 50$	0.041 ± 0.004	0.041
	$b = 100$	0.041 ± 0.004	0.041
Dedicated channels ($\alpha = 1.8, b = 20$)	$d = 0$	0.022 ± 0.002	0.019
	$d = 1$	0.041 ± 0.004	0.041
	$d = 2$	0.077 ± 0.005	0.079

The first set of simulations depicted in Fig. 2 represents the GPRS performance for different mobility factors over a wide range of GPRS offered loads ($\rho_d = 1, 2, 3, 4$). Three mobility profiles, namely, high mobility ($\alpha = 5.0$), moderate mobility ($\alpha = 1.8$) and low mobility ($\alpha = 0.2$) have been considered (Table 1, *Mobility*). As shown in these figures, both packet loss and average packet delay decrease by increasing the mobility factor. This is due to the fact that the buffer occupancy times increase as the mobility decreases, resulting in the associated increases in packet loss probability and delay.



(a) Packet loss probability.

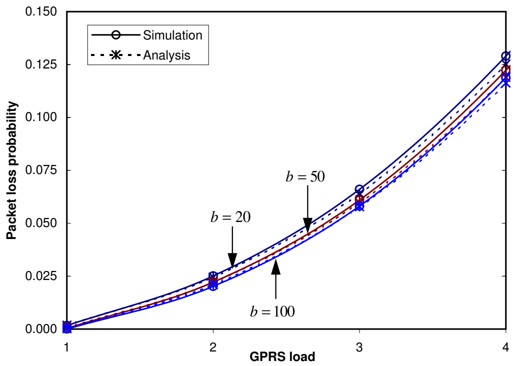


(b) Packet delay.

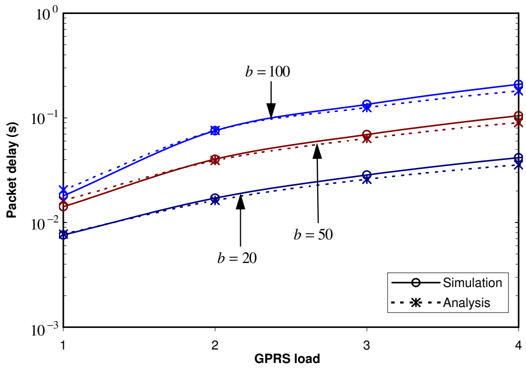
Fig. 2. Mobility effect.

The second set of simulations in Fig. 3 shows the effect of data buffer size on GPRS performance for the moderate mobility configuration (Table 1, *Buffer size*). Observed from Fig. 3(a), the packet loss probability is almost insensitive to the buffer size for the simulated range of loads. In contrast, average packet delay significantly increases by increasing the buffer size as shown in Fig. 3(b). Notice that in these simulations, there is only one dedicated data channel, thus the GPRS load of $\rho_d = 4.0$ is relatively a high load.

The third set of simulations in Fig. 4 shows the effect of dedicated data channels on GPRS performance for the moderate mobility and small buffer size configuration (Table 1, *Dedicated channels*). As expected, increasing the number of dedicated channels significantly improves the data performance. The interesting part of these figures is the case

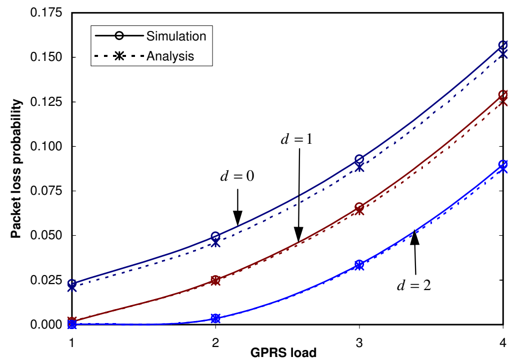


(a) Packet loss probability.

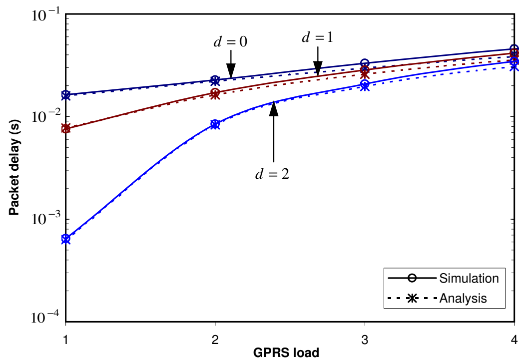


(b) Packet delay.

Fig. 3. Buffer size effect.



(a) Packet loss probability.



(b) Packet delay.

Fig. 4. Dedicated channels effect.

of $d = 0$ which shows that, in fact, GPRS service can utilize the GSM wireless resources while providing an acceptable data service for delay-tolerant applications.

Finally, as observed form all the presented simulation results, the approximate analysis provides rather accurate results for packet loss probability. However, its accuracy for average packet delay depends on the offered GPRS load. Increasing the offered load decreases the accuracy of the presented approximate analysis.

V. Conclusion

In this paper, we analyzed the performance of GPRS data service with buffering and dedicated channels for data traffic while considering the effect of reneging data packets due to the corresponding user handoff. Through analysis and simulation we showed that the impact of handoff on GPRS performance is not negligible in contrast to what is usually assumed in literature. For future work, we consider extending the presented analysis to more general cases such as non-exponential cell residence times and more realistic packet arrival processes such as Markov modulated Poisson process.

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