Max-Min Fair Capacity of Wireless Mesh Networks

Bassam Aoun and Raouf Boutaba

Abstract—**The use of WMNs as backbone for large wireless access networks imposes strict bandwidth requirements. It is therefore necessary to study and quantify the capacity of such systems. In this paper, we argue that the capacity of WMNs should be addressed in the context of fairness to ensure proper operation of WMNs. Among the fairness schemes, max-min fairness allows fair and efficient use of network resources. We therefore propose an algorithm for max-min capacity calculation, formulated in term of collision domains. In addition, we show how to calculate the effective load of collision domains, assuming IEEE 802.11 as the MAC protocol. We illustrate our proposed algorithm and validate our results over baseline and general topologies.**

Index Terms— **Max-min Capacity, Wireless Mesh Networks, IEEE 802.11**

I. INTRODUCTION

WIRELESS is well established for narrowband access systems, but its use for broadband access is relatively new. Wireless mesh architecture is a first step towards providing high-bandwidth network coverage. Mesh architecture sustains signal strength by breaking long distances into a series of shorter hops. Intermediate nodes not only boost the signal, but cooperatively make forwarding decisions based on their knowledge of the network. Such architecture provides high network coverage, spectral efficiency, and economic advantage.

Although the IEEE 802.11 MAC protocol [1] has been initially designed to operate in wireless local area networks, it has been adopted as the de facto standard for WMNs. The main reasons are its wide popularity and standardization which would enhance the scalability and inter-operability of WMNs.

Recently, interesting commercial applications of wireless mesh networks (WMN) have emerged. One example of such applications is ``community wireless networks" [2] [3]. Several vendors have recently offered WMN products. Some of the most experienced in the business are Nortel [4], Tropos Networks [5], and BelAir Networks [6]. There are more than 20 other startup companies that plan to offer similar products. Despite the recent startup surge in WMNs, much research remains to be done before WMNs realize their full potential.

The use of WMNs as backbone for large wireless access networks imposes strict bandwidth requirements. It is therefore necessary to study and quantify the capacity of such systems. In this paper, we argue that the capacity of WMNs should be addressed in the context of fairness to ensure proper operation of the WMNs. Among the fairness schemes, *max-min* fairness allows fair and efficient use of network resources. We therefore propose a framework for *max-min* capacity calculation and propose a centralized algorithm for that purpose. We formulate the capacity of WMNs in term of collision domains and use the IEEE 802.11 MAC protocol to compute the *effective* load of collision domains.

The rest of the paper is organized as follows. Section 2 discusses related work. Section 3 presents a framework to study the capacity of WMNs. Section 4 proposes an algorithm for WMNs capacity computation. Section 5 shows how to compute the effective load of collision domains. Section 6 validates our analysis through

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simulations. Section 7 concludes our work.

II. RELATED WORK

In recent years, there has been a focus on the fundamental question: *what is the maximum throughput of multihop wireless networks?*

To date and to the best of our knowledge, no study has addressed the fair capacity of WMNs operating over IEEE 802.11. Instead, previous works focused on the capacity of ad-hoc networks or did not considered the specifications of the MAC protocol.

[7] and [8] studied the capacity of ad-hoc networks from a theoretical perspective. Gupta and Kumar [7] showed that in a wireless ad-hoc network with n identical nodes, the per-node throughput is $\Theta\left(1/\sqrt{n\log n}\right)$ assuming random node placement and communication pattern and $\Theta\left(1/\sqrt{n}\right)$ for an optimal setting. Jain et al. [8] studied the influence of interference using a conflict graph, and derived upper and lower bounds on the optimal throughput. Couto et al. [9] went further and examined the interactions of the 802.11 MAC and ad-hoc forwarding.

These works considered ad-hoc networks which differ significantly from WMNs in the following aspects. First, as opposed to an ad-hoc network, WMN has a relatively stable topology except for occasional nodes failure and addition. Second, in WMN practically all the traffic is either to or from a gateway, while in ad-hoc networks the traffic flows between arbitrary pairs of nodes. The traffic inside a WMN is therefore skewed and gateways would form bottlenecks. The presence of bottlenecks affects dramatically the capacity of WMNs, making previous study on capacity of ad hoc networks not suitable.

On the other hand, Jun et al. [10] have addressed the capacity of WMNs but their work had limitations along many aspects. First, they did not study the *fair* capacity of WMNs. Second, the presented upper bound underestimates the capacity because they did not consider spatial reuse inside collision domains. Third, they considered a *single* bottleneck collision domain for the entire network, reducing the efficiency of network resources utilization.

Fairness in ad hoc networks has been addressed in various studies. While [11] [12] [13] aim at implementing max-min fairness, [14] [15] address proportional fairness. To the best of our knowledge, [16] is the only work focusing on the characteristics of WMNs. [16] defines a fairness model that addresses the requirements of multihop networks, and proposes a distributed algorithm to achieve fairness.

In this work, however, we *quantify* the fair capacity of WMNs. We leverage the properties of *max-min* fairness to propose an algorithm for capacity calculations in term of collision domains.

III. FRAMEWORK FOR WMN CAPACITY CALCULATION

A. WMN Characteristics

WMNs have a relatively stable topology except for occasional nodes failure or addition. The traffic, being aggregated from a large number of end users, changes infrequently. Practically all the traffic is either forwarded to or from a gateway.

A tree-based proactive routing scheme would easily allow flows aggregation and would minimize overhead, ensuring an optimal utilization of bandwidth [17]. A spanning tree rooted at the gateway is therefore used for traffic forwarding. Each AP's aggregate flow therefore traverses a unique and static route.

As opposed to an ad hoc network, a wireless mesh network offers predictability in term of traffic pattern. This permits capacity analysis based on "computed" traffic profiles.

B. Network Model

A WMN is represented by an undirected graph $G(V,E)$, called connectivity graph. Each node $v\in V$ represents an Access Point (AP). The *neighbourhood* of v, denoted by $\Delta(v)$, is the set of nodes residing in its transmission range. A bidirectional wireless link exists between v and every neighbor $u\in\Delta(v)$ and is represented by an edge $(v,u)\in E$.

$G_R(V,E_R)$ represents the spanning tree rooted at

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the gateway and used for proactive routing. E_R is a subset of E used to route traffic flows and therefore carries data. On the other hand, the subset $E_I = \{E - E_R\}$ is not used for traffic forwarding and therefore carries no data. E_I is however useful to convey information indicating interference.

Each AP $v \in V$ aggregates the traffic from mobile clients in the coverage range of its access link. The aggregated traffic would form a flow f_v being forwarded to the gateway; f_v has a constant value representing the throughput of the aggregated flow generated at AP v.

We introduce the binary variable $A_{v,e} \in \{0,1\}$ to indicate whether the flow f_v of $v \in V$ traverses the wireless link $e \in E_R$ along its route to the gateway. The load l_e of a wireless link $e \in E_R$ is the sum of all flows f_v traversing e and is calculated as follows: $l_e = \sum_{v \in V} A_{v,e} \cdot f_v$. We note that $(\forall e \in E_R)$ $l_e > 0$, and $\forall e \in \{E - E_R\}$ $l_e = 0$.

C. Wireless Interference and Collision Domains

We recall that, in a wireless network, the resource of interest is not a link but a wireless channel. Neighboring wireless links contend and share the capacity of the local channel. The contention experienced by each wireless link is represented by the load of its collision domain, defined as follows:

Definition 1 (Collision Domain): The collision domain C_e of a wireless link $e \in E$ is the set of neighbouring wireless links that share its local channel, and therefore interfere with its transmission. From a protocol point of view, the collision domain C_e of a wireless link is the set of neighbouring wireless links that have to be inactive in order for the wireless link to transmit successfully.

For convenience, we include the wireless link e itself in C_e, such that the total load carried by the links in C_e is limited by the capacity W of the local channel.

D. Fairness Consideration

Capacity of a WMN should be addressed in the context of fairness to ensure proper operation of WMNs. The targeted granularity of fairness is an AP-aggregated flow f_v. In particular, each AP corresponds to a single residence, small business, or hot spot, and this AP traffic f_v should be treated as a single aggregate, independent of the number of TCP micro-flows or mobile devices supported by that AP.

Aggregate flows f_v for $\forall v \in V$ should be treated equally independent of the relative location of v with respect to the gateway; users should not be penalized for not having a nearby wireline Internet connection (i.e. a gateway). Such fairness mechanism is opposed to the *capacity-maximizing* allocation consisting of starving multi-hop flows and giving all the capacity to one-hop flows [18] [19].

In addition to fairness, we would like to achieve efficient use of network resources. That is, network resources are to be reclaimed by other flows, when they are unused by flows bottlenecked elsewhere.

E. Fairness Reference Model

Among the fairness mechanisms, *max-min* fairness allows the fair and efficient use of network resources and is widely considered in wired [20] and wireless networks [21] [22]. A more general framework, namely traffic allocation, is considered in the fields of operations research [23].

In our study, we assume that *max-min* fairness is enforced among APs' aggregate flows f_v. We recall that vector $\vec{f}$ is defined such that the i^{th} coordinate is the rate allocation for AP $i \in V$ aggregate flow. We assume that all transmission rates are positive. We present the definition of *max-min* fairness as follows:

Definition 2 (Max-min Fairness): [18] An allocation of rate $\vec{f}$ is *max-min* fair if and only if an increase of any rate *i*, within the domain of *feasible* allocations, must be at the cost of a decrease of some already smaller rate. Formally, for any other rate allocation $\vec{y}$, if $y_i > f_i$ then there must exist some $j \in V$ such that $f_j < f_i$ and $y_j < f_j$.

The rate vector $\vec{f}$ is to be calculated, and the sum of its components $\sum_{i\in V} f_i$ is considered the total fair capacity of the WMN.

A *feasible* allocation of rates $\vec{f}$ is such that for $(\forall e\in E_R)\ \sum_{w\in C_e} l_w \le W \Rightarrow \sum_{w\in C_e}\sum_{v\in V} A_{v,w}\cdot f_v \le W$. That is, for $(\forall e\in E_R)$, the total load carried by the wireless links in collision domain C_e is limited by the channel capacity W.

IV. MAX-MIN CAPACITY CALCULATION

A. Max-min Capacity Properties

We introduce $N(C,F)$ to represent the network resource model of the WMN. F is the set of APs aggregate flows f_v, $|F|=|V|$. We drop the sub-index v from f_v to represent flows independently from the AP $v\in V$ where they originate. C is the set of collision domains C_e of wireless links $\forall e\in E_R$, $|C|=|E_R|$. Similarly, we drop the sub-index e from C_e to represent collision domains independently of the wireless link e they are associated with.

We start with the following theorem. The detailed proof, in the context of wired networks, can be found in [20].

Theorem 1: A feasible allocation of rates $\vec{f}$ is max-min fair if and only if every flow has a bottleneck collision domain.

The key concept of a *bottleneck* collision domain is defined as follows:

Definition 3: A collision domain $d\in C$ is a bottleneck for flow $f\in F$ if and only if

1. Collision domain d (a subset of E_R) is saturated; that is $\sum_{e\in d}\sum_{f\in F} A_{f,e}\cdot f = W$.
2. and, flow f has the maximum rate among all flows traversing collision domain d: $f \ge f'$ for all $\{f'\in F \mid A_{f',e}=1 \text{ for } \forall e\in d\}$.

A collision domain d is *saturated* if the total load on its links is equal to the full capacity W of the wireless channel. We recall that $\sum_{f\in F} A_{f,e}\cdot f$ is the load on the wireless link e.

Intuitively, a *bottleneck* collision domain for flow f is a collision domain which is limiting, for the given allocation.

B. Algorithm for Capacity Calculation

In order to ensure max-min fairness, we start by identifying the collision domain with the *smallest capacity available per flow*. Such collision domain is denoted as the *bottleneck collision domain of the network* $N(C,F)$.

Definition 4: Collision domain $d\in C$ is called bottleneck with respect to the network $N(C,F)$ if

$$\frac{Cap_d}{\sum_{e\in d}\sum_{f\in F} A_{f,e}} = \min_{s\in C}\frac{Cap_s}{\sum_{e\in s}\sum_{f\in F} A_{f,e}}$$

where $\sum_{e\in d}\sum_{f\in F} A_{f,e}$ is the nominal load of the collision domain, and Cap_d is the remaining available capacity of collision domain d. For the original network resource model $N(C,F)$, Cap_d is equal to W.

We now present the iterative "network bottleneck identification" method to allocate flows' rate, achieving max-min fairness. First, we identify all "bottleneck" collision domains of $N(C,F)$. We share the capacity of each collision domain equally among all flows traversing it. Then we remove these flows from the network, and reduce the capacity of every collision domain by the bandwidth consumed by the removed flows crossing it. The resulting network model is therefore reduced to $N^1(C^1,F^1)$, where F^1 is the remaining flows and C^1 is the updated capacity of collision domains. We now identify the *next level* bottleneck collision domain of the reduced network $N^1(C^1,F^1)$ and repeat the procedure. We continue until all flows are assigned rates.

Theorem 2: The iterative "network bottleneck identification" method to allocate flows rates is *max-min* fair.

Proof: We will prove this theorem by satisfying the two conditions of *Definition 3* for each flow. Since at each iteration the entire capacity of the bottleneck collision domain $d\in C$ is shared among flows F crossing it, d is saturated. This satisfies condition 1 of *Definition* 3. In addition,

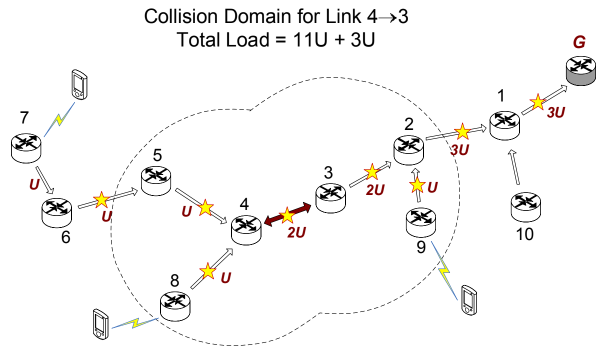

Figure 1. Illustration of a collision domain

since bottleneck collision domains are chosen iteratively, in the order of smallest capacity available per flow, condition 2 of *Definition* 3 is satisfied. Therefore, upon the completion of the *iterative network bottleneck* algorithm, each flow has already crossed a bottleneck collision domain. Given *Theorem* 1, we have proved that the capacity calculation algorithm is *max-min* fair.

Intuitively, it can be seen that the rate allocation obtained in such a way is *fair* in the sense that all flows constrained by a particular bottleneck get an equal share of this bottleneck capacity. It is also *efficient* in the sense that given a fair allocation, no more data can be pushed through the network, since each flow crosses at least one fully *saturated* channel.

C. *Load of Collision Domains*

We used so far the *nominal* load of collision domains, represented by $\sum_{e\in d}\sum_{f\in F} A_{f,e}$, to calculate the capacity available per flow (see *Definition 4*). We used the *nominal* load for simplicity of illustration. However, the capacity available per flow is more accurately calculated using the *effective* load of collision domains which is more challenging, assuming in particular IEEE 802.11 as the MAC protocol.

V. EFFECTIVE LOAD OF COLLISION DOMAINS

A. *Wireless Links Constituting Collision Domains*

In order to compute the effective load of a collision domain C_e, we first identify the set of wireless links forming C_e; that is, the set of neighboring wireless links that have to be inactive in order to the wireless link e to transmit successfully. Those links are identified using: a) the *coordinated channel access* imposed by IEEE 802.11 standard, and b) the *uncoordinated channel access* still present in multi-hop scenarios.

1) Imposed by Coordinated Channel Access

Although the broadcast nature of the wireless medium implies that *no receiving node can be in the reception range of more than one simultaneously transmitting node*, the IEEE 802.11 standard imposes more strict constraints on channel access in order to mitigate the “exposed” and “hidden” node problems [24]. The MAC layer has to ensure that *no node that is a one-hop neighbor of either the sender or the receiver of a data packet may be engaged in any communication activity (either transmitting or receiving) during the entire 4-way (RTS-CTS-DATA-ACK) exchange.*

Figure 1 illustrates the collision domain $C_{4\to3}$ of the link $4\to3$. There are 3 flows in the network, generated at nodes 7, 8 and 9. Flows are aggregated and forwarded towards the gateway. The two semi-circles contain the nodes that are one-hop neighbor of either the sender or the receiver. Due to the *coordinated channel access* imposed by the IEEE 802.11, the collision domain includes all the wireless links included in or intersecting the two semi-circles.

2) Imposed by Uncoordinated Channel Access

The hidden node problem still exists in multihop networks, although the standard has paid much attention to this problem. The proposed RTS/CTS handshaking and carrier sensing work well to prevent the hidden node problem in a WLAN where all nodes can sense each other's transmission [25]. Obviously, this is not always true in a multihop network, as shown in Figure 2a.

The sending nodes 1 and 4 are outside each other sensing range, making carrier sensing ineffective. This situation leads to *uncoordinated channel access*, starving flow $1\to2$ for the following reasons. Since node 1 can not sense the transmission of node 4, it will continuously try to access the channel. However, node 2 can not reply to the RTS, making the backoff timer of node 1 to increase exponentially. Moreover, in case node 1 succeeded with the RTS/CTS exchange, subflow $4\to G$ transmission would interfere with subflow

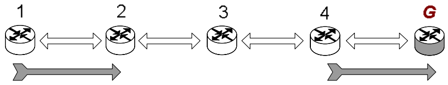

Figure 2a. Hidden Node Problem: Topology

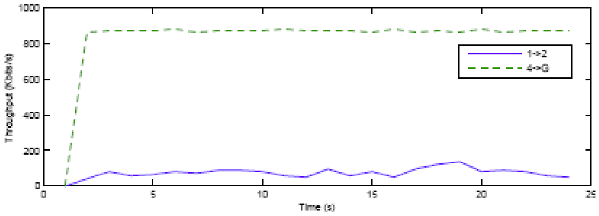

Figure 2b. Hidden Node Problem: Flows' Throughput

$1 \rightarrow 2$, causing collisions at node 2.

Figure 2b shows the starvation of flow $1 \rightarrow 2$ when the loads offered at node 1 and node 4 are equal to the full capacity of the MAC layer (refer to Section VI-A for experimental details).

Therefore, in addition to the constraints imposed by the IEEE 802.11 standard in Section V-A.1, the collision domain should also include link $1 \rightarrow G$ which introduces a hidden node problem, similar to the scenario illustrated in Figure 2a.

B. *Nominal vs. Effective Load of Collision Domains*

Every collision domain is bounded by the capacity of the MAC layer (wireless channel) and should be able to forward the traffic of its links.

From Figure 1, we observe that the total traffic to be forwarded inside the collision domain is 14U composed of 11U as a result of the coordinated channel access (RTS/CTS), and an additional 3U due to the hidden node problem, where U is the unit of traffic we shall compute. The *nominal* load of the collision domain $C_{4\rightarrow 3}$ is therefore 14U, consisting of the entire traffic forwarded by the wireless links in $C_{4\rightarrow 3}$.

The location-specific nature of contention, coupled with the multi-hop nature of the network, allows for *spatial channel reuse*. Any subflows that are not interfering with each other can potentially transmit simultaneously. Therefore, the amount of traffic to be forwarded individually by the collision domain is *less than or equal* to the sum of the traffic on its links. Simultaneous transmissions should be considered, and deducted from the total load on the channel. The work by [10] did not consider spatial reuse which invalidates the proposed upper bound on the capacity.

In Fig 1, we can see that link $6 \rightarrow 5$ can transmit simultaneously with link $2 \rightarrow 1$ and $1 \rightarrow G$. Similarly, link $5 \rightarrow 4$ and link $8 \rightarrow 4$ can transmit simultaneously with link $1 \rightarrow G$. To account for spatial reuse, we remove the load of the least congested link among the pair of simultaneously transmitting links (eliminating double counts).

We therefore remove 1U for each of the following simultaneous transmissions: $(6 \rightarrow 5, 2 \rightarrow 1)$, $(5 \rightarrow 4, 1 \rightarrow G)$ and $(8 \rightarrow 4, 1 \rightarrow G)$, for a total of 3U. The *effective* load of the collision domain is therefore reduced to 11U, sharing the capacity of the MAC layer W.

VI. VALIDATION AND SIMULATION RESULTS

A. *Experimental Setting*

NS-2 with CMU wireless extensions [26] is used for simulations. The parameters are tuned to the commercially available 802.11-based WaveLan wireless cards. The effective transmission range is 250 meters and the sensing (interference) range is about 550 meters. The simulations involve nodes separated by 200 meters, which allows a node to connect only to neighboring nodes. The bandwidth is set to 1 Mbps and RTS/CTS exchange precedes all data packets. NS-2 is extended to support static routing, more adapted for WMN environments; we therefore eliminate overheads due to routes discovery and maintenance.

To validate our results, we first need to calculate the effective MAC capacity W. This is done by calculating the throughput in one hop scenario. Although the nominal MAC data rate is set to 1 Mbits/s, we get W = 876 Kbits/s. The difference is due to MAC layer overheads consisting of exchanging preamble for synchronization and performing RTS/CTS handshakes.

B. *Methodology to Ensure Max-min Fairness*

To validate the proposed algorithm, we compare the computed capacity against the *max-min* fair capacity obtained in our simulations. To evaluate in our simulations the *max-min* fair capacity of networks operating over IEEE 802.11 protocol, we propose an approach called *progressive filling*.

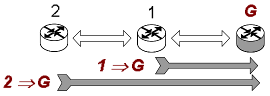

Figure 3a. Starvation: Simultaneous Flows

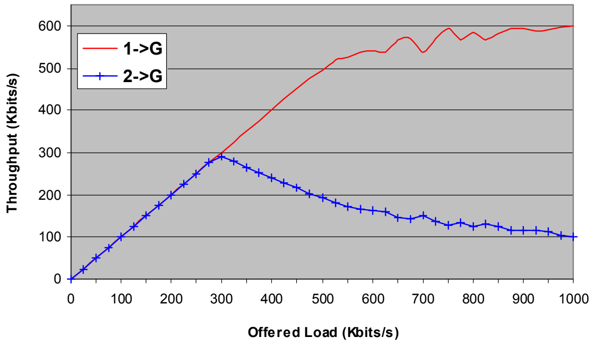

Figure 3b. Starvation: Throughputs vs. Offered Load

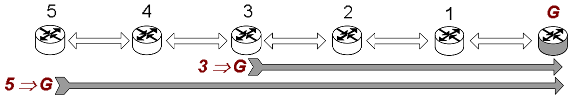

Figure 4a. 2 Exclusive Flows

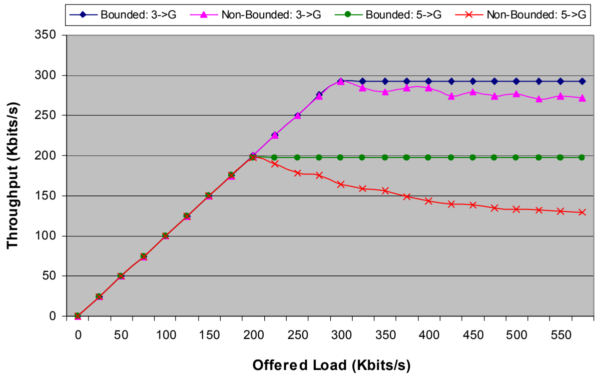

Figure 4b. Bounded vs. Non-Bounded Load Offering. Degradation of throughputs as load exceeds the fair capacity.

The approach consists of starting all offered rates at 0 and growing all rates together at the same pace. At some point, the total traffic would exceed the capacity of the wireless channel, forming a bottleneck. Beyond that point, unfairness would result: the throughputs of less favored flows will decrease while others continue to increase. The rates of those inter-dependent flows that formed a bottleneck are not increased anymore, and we continue increasing the rates of other flows. The procedure terminates when no flow can increase its throughput further without reducing less privileged flows' throughput.

The *progressive filling* approach is necessary because the IEEE 802.11 protocol fails to ensure fairness when the offered loads exceed the available capacity in the network, even in simple scenarios as shown in Figure 3. The unfairness illustrated in Figure 3b is a result of the imprecise-EIFS problem [27]. Applying *progressive filling* to the scenario of Figure 3a, *max-min* fair throughputs correspond to offered loads of 292 Kbits/s. The maximum load offered to each flow is therefore restricted to its maximum fair capacity; hence more privileged flows would not have the opportunity to starve others.

The enforced maximum throughput of 292 Kbits/s can also be derived by our proposed algorithm, as follows. Flows $1 \to G$ and $2 \to G$ cross a common bottleneck collision domain of 3U, where U is the unit of traffic to be computed. The capacity of the MAC layer 876 Kbits/s is shared among those flows, leading to U=876/3=292 Kbits/s. Referring to Figure 3b, 292 Kbits/s is indeed the maximum attainable fair throughput obtained by the *progressive filling* approach.

In addition to enhancing fairness between different flows, *progressive filling* is also appropriate when considering a single flow. Bounding the maximum offered load at a source node helps reducing packet losses (i.e. dropping) and prevents wasting bandwidth. IEEE 802.11 fails to achieve the optimum chain schedule because the node's ability to send is affected by the amount of competition it experiences. For example, a node at the beginning of a chain could actually inject more packets than the subsequent nodes can forward. These packets are eventually dropped at later nodes. The time this node spends sending those packets decreases the delivered throughput since it prevents transmissions at subsequent nodes. Figure 4 shows that the throughput degrades as the offered load exceeds the network capacity. The threshold obtained using *progressive filling* can also be derived using our proposed algorithm, as shown in the next section.

Restricting the offered load to the flow's maximum fair capacity would therefore ensure fairness and avoid wasting bandwidth.

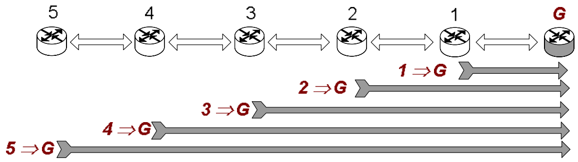

Figure 5a. Chain Topology

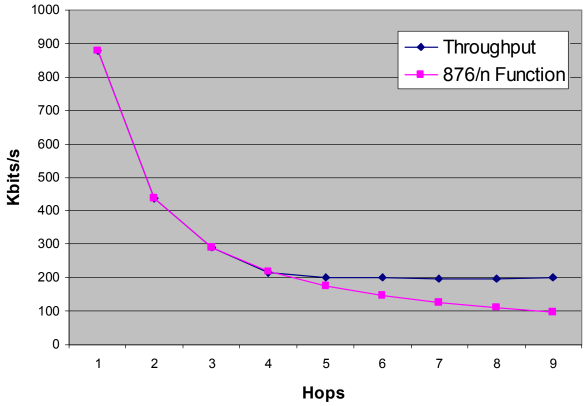

Figure 5b. Scalability of Throughputs over a chain topology

C. Baseline Scenario

1) Single-Flow Chain Topology

In this section, we consider the throughput of *single-flow* chain topologies. We consider the flows in Figure 5a operating individually.

As we increase the number of nodes along the path to the gateway, packets are forwarded additional hops. Therefore, we expect flows throughput to scale as 876/n, where n is the number of nodes along the path to the destination.

Figure 5 plots the obtained throughput versus 876/n. From 1 to 3 hops, channel access is effectively coordinated by the RTS/CTS exchange. The channel capacity W is therefore shared perfectly among the three subflows, as shown in Figure 5b; the throughput matches exactly 876/n. However, in the case of 4 hops, we see that there is a slight degradation of throughput compared to 876/4. This is due to collisions caused by the *forward hidden node* problem, as explained in SectionV-A.2. Finally, we can clearly see that the throughput reaches a steady state around 200Kbits/s beyond 4 hops, indicating that the bottleneck reached a maximum and constant size. On a chain topology longer than 4 hops, the effective load is 5U-U=4U, accounting for spatial reuse. Theoretically, flows' throughput should be around U=876/4=219 Kbits/s.

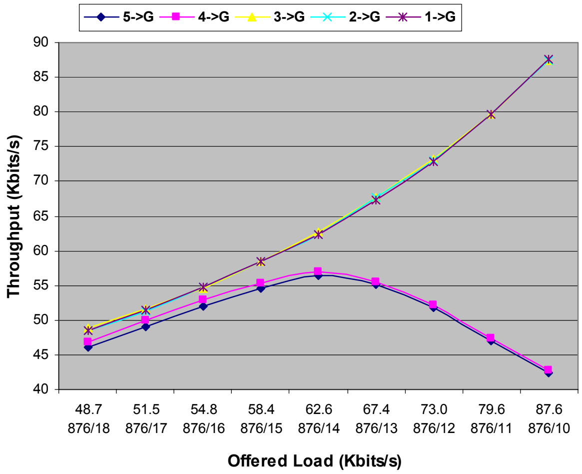

Figure 6. Throughputs versus offered loads of the 5 simultaneous flows of Figure 5.

Ideally, the throughput should converge to 219 Kbits/s but it is reduced to 209 Kbits/s for 4 hops then to 199 Kbits/s for 5 or more hops. The reason is that at 4 hops there is only one hidden node, reducing the throughput by 10 Kbits/s and there is an additional hidden node in the collision domain for 5 or more hops, hence reducing the throughput another 10 Kbits/s, to settle down at 199 Kbits from its ideal 876/4 Kbits/s.

We note that the throughput is always higher than 876/5 =175.2Kbits/s which does not account for spatial reuse, invalidating the proposed upper bound on the throughput given by [10].

2) Multi-flows Chain Topology

In this section, we consider all five flows in Figure 5a operating simultaneously. From Figure 5a, we can calculate the traffic load on every collision domain.

The collision domain $C_{3\to 2}$ is most congested and consequently forms a bottleneck. $C_{3\to 2}$ has a *nominal* traffic load of 15U and an *effective* traffic load of 14U after deducting the load of link $5 \to 4$ since it can forward simultaneously with link $1 \to G$, due to spatial reuse. We therefore assign to all flows crossing $C_{3\to 2}$ a maximum fair throughput of 876/14=62.6 Kbits/s.

Figure 6 plots the throughputs of all flows using the *progressive filling* approach. Limiting the maximum offered load to 62.6 Kbits/s is necessary, because if we allow nodes to send at a higher data rate, they will gradually starve the flows $f_{5\to G}$ and

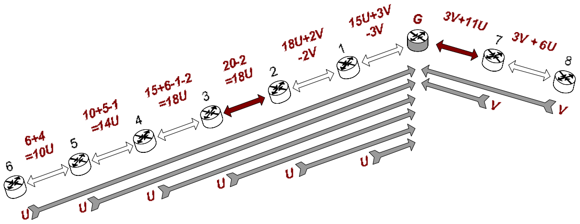

Figure 7. Multipath Topology: 2 Paths Scenario

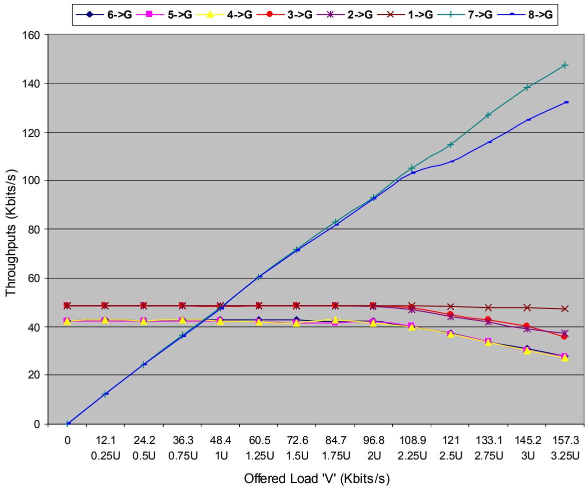

Figure 8. Multipath Topology: Throughputs as V increases

$f_{4\rightarrow G}$. We can see in Figure 6 that 62.6 Kbits/s, derived from the computed collision domain load of 14U, is indeed the optimal load to offer across each flow to preserve a fair throughput.

Unfortunately, $f_{5\rightarrow G}$ and $f_{4\rightarrow G}$ can not reach the maximum fair throughput due to the hidden node problem at links $5\rightarrow 4$ and $4\rightarrow 3$, causing collisions. This problem can not be eliminated even if we underuse the channel (i.e. offered load <62.6 Kbits/s), due to the exponential backoff of the 802.11 standard.

D. General Topology

In this section, we study the throughput of a multi-path topology. Figure 7 presents a simple scenario for illustration, consisting of two paths to the gateway. The *effective* load of collision domains is shown in Figure 7, calculated as explained in Section V.

For example, if we consider link $4\rightarrow 3$, the collision domain consists first of 15U, accounting for 802.11 RTS/CTS channel constraints. We add 6U to account for the hidden node problem due to the link $1\rightarrow G$. Next, we subtract 1U and 2U to account for spatial reuse of links $6\rightarrow 5$ and $5\rightarrow 4$ respectively, which can transmit simultaneously with link $2\rightarrow 1$ and $1\rightarrow G$.

We first identify the collision domain with the *smallest capacity available per flow*. Referring to *Definition 4*, $C_{3\rightarrow 2}$ is the first bottleneck collision domain of the topology shown in Figure 7. It has an effective load of 18U. The maximum achievable fair throughput of all flows crossing $C_{3\rightarrow 2}$ is therefore U=876/18=48.7 Kbits/s. Flows crossing $C_{3\rightarrow 2}$ (i.e. crossing any link 1 in $C_{3\rightarrow 2}$ are $f_{6\rightarrow 5}$, $f_{5\rightarrow 4}$, ... $f_{1\rightarrow G}$, consisting of all the flows to the left of the gateway.

We therefore remove those flows from the network and reduce the collision domains by the amount consumed by any of those flows. For example, we consider the load of the collision domain $C_{7\rightarrow G}$ consisting of 3V+11U=W. We subtract from W (876 Kbits/s) the amount consumed by 11U. The capacity of $C_{7\rightarrow G}$ is therefore reduced to $876-11\times 48.7=340.3$. Similarly, the capacity of $C_{8\rightarrow 7}$ is reduced $876-8\times 48.7=486.4$.

Among the remaining collision domains $C_{8\rightarrow 7}$ and $C_{7\rightarrow G}$, $C_{7\rightarrow G}$ is the bottleneck since it has the *smallest capacity available per flow* (V=340.3/3). The *max-min* fair throughput of $f_{8\rightarrow G}$ and $f_{7\rightarrow G}$ is therefore V=340.3/3=113.4 Kbits/s.

We use *progressive filling* approach to find flows throughput and validate our theoretical results. We first increase *all* offered loads starting from 0. The offered load of $f_{6\rightarrow 5}$, $f_{5\rightarrow 4}$, ... $f_{1\rightarrow G}$ is stopped at 48.7 Kbits/s, since unfairness appears after that (similar to Figure 6). On the other hand, the throughputs of $f_{8\rightarrow G}$ and $f_{7\rightarrow G}$ is further increased. Figure 8 illustrates the second step. It plots the throughputs of all flows as V (the throughput of $f_{8\rightarrow G}$ and $f_{7\rightarrow G}$) increases from 0 to 157 Kbits/s. We can clearly see that V only affects other flows' throughput when it reaches a value between 108.9 Kbits/s (2.25U) and 121 Kbits/s (2.5U), which validates the theoretical value 113.4 Kbits/s calculated above.

Therefore, the maximum fair capacity of U and V is 48.7 Kbits/s and 113.4 Kbits/s respectively.

This constitutes a higher throughput than the upper bound on the fair capacity provided by [10] which allocates 48.7 Kbits/s for all flows, considering one bottleneck for the entire network.

VII. CONCLUSION AND FUTURE WORK

In this paper, we argued that the capacity of WMNs should be addressed in the context of fairness to ensure proper operation of the WMNs. Among the fairness mechanisms, *max-min* fairness allows fair and efficient use of network resources. We therefore studied *max-min* fair capacity in the context of WMNs and proposed an algorithm for capacity calculation, formulated in term of collision domains.

Next, we showed how to calculate the effective load of collision domains, assuming IEEE 802.11 as the MAC protocol. We first identified the wireless links constituting the collision domain derived from the *coordinated* and *uncoordinated* channel access. Second, we identified spatial reuse and showed that the *effective* load on a collision domain is less than or equal to the sum of the traffic on its links.

We assumed throughout this work a constant effective MAC capacity W. However, it is hardly the case in reality, due to environmental interference beyond our control. In addition, varying link quality, between pairs of nodes, has not been considered. To account for those stochastic factors, a more sophisticated scheme would include decentralized MAC capacity measurement mechanisms.

The followings are possible directions for future work. First, it would be interesting to study improvements on the capacity when considering directional antennas. Second, implementing a decentralized resource management algorithm to compute the fair capacity of APs is essential for admission control.

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