Totally Disjoint Multipath Routing in Multihop Wireless Networks

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Abstract—Spreading traffic over multiple paths has been shown to enhance network performance compared to single path routing. However, the route coupling effect specific to wireless environments (consequent to the shared transmission medium) can significantly reduce the benefits of such an approach. In this paper, we focus on the 2-path routing problem in a non-mobile and non energy-constrained network. We evaluate the network performance and the effect of interference in a single source-destination pair scenario and for multiple source-destinations pairs. In the former case, we provide an analytical evaluation of the throughput and propose a position-based algorithm to iteratively find a path between a source and a destination. In the latter case, we demonstrate how interference can significantly degrade the nominal network capacity and void the benefits of multipath routing.

I. INTRODUCTION

Multipath routing is promoted as an alternative to single path routing for its potential to address issues such as route failure and recovery, and network congestion. In energy constrained networks such as ad hoc networks and sensor networks, it has been demonstrated that multipath routing achieves higher performance than single path routing resulting in an overall lower power consumption [2] [11].

Recently, proposals for multipath routing in wireless networks have flourished, the main contributions focusing on either improving path reliability through the establishment of backup paths [8] [12] [16] or on optimizing network resource utilization via load distribution among several paths [9] [14]. Various path selection strategies have been proposed with differences in the parameters to optimize (packet drop ratio, overhead, or end-to-end delays), the level of fault tolerance required, or the congestion avoidance strategy (link-disjoint vs. node-disjoint paths). Several multipath routing approaches consisted in adapting prominent single path routing protocols such as AODV and DSR [9] [13] for multiple paths routing, by modifying the forwarding process of resource requests and the information maintenance at intermediary nodes [1] [10] [20].

These works have considered some (or all) of the constraints pertaining to ad hoc networks: node mobility, energy limitations, high network density and limited computation capabilities. However, only a few works have actually considered the effect of interference in multipath routing and, to the best of our knowledge, none has provided an analytical evaluation of the network performance in this context.

In this work, we provide an evaluation of the throughput in a multipath routing strategy while taking into consideration the impact of interference. We focus on networks with fixed and non-energy constrained wireless backbone, with potentially enhanced capabilities such as GPS systems. We adopt an incremental approach to address this problem by first considering only the interference between a same source-destination pair and next, between multiple sources and destinations.

The remainder of the paper is organized as follows. In Section II, we describe the challenges of multipath routing in wireless environments. We present the 2-path routing problem in Section III. In Section IV, we provide an analytical evaluation of the network throughput for the single source-destination scenario and focus on the multiple source-destination pairs case in Section V. Section VI concludes this paper.

II. MULTIPATH ROUTING IN MULTIHOP WIRELESS NETWORKS

A. Route coupling

In wireless communications, severe performance degradation can result from interference of concurrent data transmissions. The shared transmission medium constrains all nodes in the interference range of a sender or receiver to inactivity until completion of the ongoing communication. When adopting a multipath strategy, this problem is further exacerbated as, in order to increase the nominal throughput beyond what single path routing would offer, it is necessary to guarantee that the chosen paths do not interfere with each other. The set of paths between a source node \( s \) and a destination node \( t \) can be of two sorts:

- link-disjoint paths: one communication link can not be shared by several paths.
- node-disjoint paths: a same node can not be part of several paths. To account for the potential interference between concurrent data transmissions, it is important to distinguish the two following subcases, in which distinct paths can be:

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1) edge-connected and therefore interfere. This phenomenon of interference between two (or more) paths is known as route coupling.
2) 0-edge-connected and do not interfere. This latter case is referred to as totally disjoint paths in the remainder of this paper.

To evaluate the impact of such topologies on the maximal achievable throughput, let us consider that a source node and a destination node are 6 hops away, with a nominal network capacity of $B$ (throughput that can be achieved at the MAC layer in one hop [5]). In the example considered, when the paths are totally disjoint, the throughput is limited to $2B/5$ (Fig. 1a) as the entire path is in the interference range of the middle link (represented in bold in Fig. 1). When the paths are node-disjoint but are 1-edge connected, the maximal throughput is bounded by $2B/7$ (Fig. 1b). This throughput is further reduced to $2B/9$ when a node is shared by 2 paths (Fig. 1c).

B. Related Works

In [17], the authors emphasized the impact of route coupling effect, which occurs when two paths are located close enough to interfere. They proposed a framework for multipath routing in ad hoc networks using directional antennas. Control packets are exchanged in omni-directional mode whereas data packets are transmitted in directional mode, therefore allowing multiple transmissions at the same time. This mechanism appears nonetheless insufficient to prevent data collision as control packets are broadcast and can conceivably interfere with any ongoing data communication occurring in the vicinity of the sender or the receiver. Therefore the usefulness of directional antennas in this context is disputable. Inter-path interference has also been considered in [19], in which the authors proposed a path selection mechanism based on the interference level between two paths (for a given source-destination pair), determined empirically via measurements.

Few works have tried to derive an analytical formulation of the performance of multipath and single path routing algorithms in multi-hop wireless networks. In [15], the authors assumed that, when the network density is high enough, routes can be approximated by straight lines. Therefore, by computing the number of routes going through a particular node, the relayed traffic can be evaluated and the number of queued packets can be computed. A similar approach is used for multipath routing but without specifically considering the number of routes per connection.

[3] improved over the previous work by alleviating the results obtained by [15] from the assumption of uniform load distribution. The authors considered that in k-shortest paths routing the routes between a source and a destination are contained within a rectangular area and, given a destination node and a relay node, the position of the possible source nodes can be determined. All the nodes satisfying these conditions are assumed to be part of a path going from the source to the destination.

But these two works did not take into consideration the problem of interference, limiting their applicability to wireless networks.

III. TOTALLY-DISJOINT 2-SHORTEST ROUTING

A. Problem Definition

In this paper, we focus on a special case of multipath routing: 2-path routing.

Definition Given a directed graph $G(V, E)$ and two nodes $(s, t) \in V$, 2-path consists of finding two paths $P_1$ and $P_2$ between $s$ and $t$ such that: a) all the nodes in $P_1$ and all the nodes in $P_2$ form a connected graph; and b) there exists no edge between any node in $P_1$ and any node in $P_2$.

B. Assumptions and Design Decisions

In the remainder of the paper, the following assumptions and design decisions have been made. First, the nodes are uniformly distributed with a density $\rho$ in an area of radius $R$. For simplification, the interference range is assumed to be the same as the transmission range (this does not impact the computation but simplifies the problem formulation).

We further consider that the network density is high enough to approximate the shortest path between any source-destination pair by a straight line. Therefore, the 2-path problem can be reduced to abstracting a path as a trajectory and to determining the probability that a node is located on it.

As we focus on totally-disjoint 2-shortest paths, the solution paths $P_1$ and $P_2$ should be spaced by at least a distance of $r$ (interference range) to prevent the route coupling effect as previously described. As we are looking for the shortest paths, the sets of possible solutions are therefore located in the bands of width $\epsilon$ (whose computation is further described).

IV. THROUGHPUT ESTIMATION: THE SINGLE SOURCE-DESTINATION SCENARIO

A. Multipath routing vs. Single path routing

To estimate the benefits of multipath routing over single path routing, we run several simulations in the ns-2.28 network simulator [4]. IEEE 802.11b is used as the underlying MAC protocol and UDP as the transport protocol. The routing is static and no overhead due to route establishment is accounted for. The network is composed of a source node and a destination node located 6 hops apart (Fig. 2). We evaluate the throughput for a single path strategy, a multipath strategy with totally disjoint paths and a multipath strategy with interfering paths (the paths are 100m apart). A single flow is sent from Node 0 to Node 6. The simulation parameters are summarized in Table I.
Fig. 1. Collision domains (shadowed areas)

![Fig. 1](image1)

(a) without connection  
(b) with link connection  
(c) with node connection

Fig. 3. Throughput

![Fig. 3](image2)

(a) Single Path  
(b) Interfering 2-path  
(c) Non-interfering 2-path

<table>
<thead>
<tr>
<th>Datarate</th>
<th>11Mbps</th>
</tr>
</thead>
<tbody>
<tr>
<td>Packet size</td>
<td>1500 bytes</td>
</tr>
<tr>
<td>Transmission range</td>
<td>250m</td>
</tr>
<tr>
<td>Traffic</td>
<td>CBR at 0.005 packets/s</td>
</tr>
<tr>
<td>Transport Protocol</td>
<td>UDP</td>
</tr>
<tr>
<td>Routing Protocol</td>
<td>Static</td>
</tr>
</tbody>
</table>

TABLE I  
SIMULATION PARAMETERS

Fig. 3 represents the throughput measured at the destination node. We can observe that the highest performance is achieved in the multipath scenario without interference (Fig. 3c), whereas the performance significantly decreases when interference is introduced and with single path routing. This result corroborates the necessity of using non interfering paths when a multipath routing strategy is adopted.

B. Throughput Estimation: Methodology

Although we are focusing in this paper on the evaluation of the throughput in a totally-disjoint 2-shortest path problem, a similar method can be applied to address the k-shortest paths routing problem.

Let A and B be a source-destination pair and F a node part of a path between A and B. The evaluation of the throughput at F necessitates to evaluate the traffic generated by the node itself and the traffic forwarded by F but generated by nodes geographically distant from F.

Therefore, the throughput computation can be broken into two steps:

- Step 1: Determine the maximum number of paths going through node F.
- Step 2: Determine the probability of participation of Node F in the forwarding process. Indeed, being geographically located on the trajectory between A and B does not necessarily imply a node participation in the forwarding process. For instance, if the communication between node A and node B necessitates 4 hops, only 3 relay nodes are needed, although more nodes (depending on the network density) can be potential candidates.

C. Detailed Analysis

1) Relay Traffic: To compute the number of paths going through a particular node F, we need to locate the source and destination nodes that can potentially have a path going through F. The method consists in computing the tangents to the circle centered at F of radius r/2. The tangents and their parallel located at δ define an area in which the source and destination node should be located.

The equation of the circle $C_F$ centered at F with radius $r/2$ can be straightforwardly derived as:

$$x^2 + y^2 - 2xx_F - 2yy_F + x_F^2 + y_F^2 - (r/2)^2 = 0$$  \hspace{1cm} (1)

The equation of the tangent to $C_F$ at $P_1(x_1, y_1)$ is:

$$xx_1 + yy_1 - x_F(x + x_1) - y_F(y + y_1) + x_F^2 + y_F^2 - (r/2)^2 = 0$$  \hspace{1cm} (2)

The coordinates of the intersection points $A(x_A, y_A)$ and $B(x_B, y_B)$ between the tangent and the circle $C_O$ centered at 0 can be determined from Eq. 1 and Eq. 2. For each point $P_1 \in$
\( C_F \), we can therefore compute the area where the possible source and destination nodes are located, and by consequence the maximal number of paths going through \( F \) at \( P_1 \). We obtain:

\[
N_{\text{max}}(F, P_1) = \sqrt{(x_A - x_1)^2 + (y_A - y_1)^2} \times \sqrt{(x_B - x_1)^2 + (y_B - y_1)^2} \epsilon \times \rho^2
\]  

(3)

From this, we need to remove the paths whose length is less than \( r \) (meaning that the source node and destination node are in direct transmission range of each other). This can be derived by applying Crofton’s formula [18]. Let us consider \( n \) points \( \xi_1, \ldots, \xi_n \) randomly distributed on a domain \( S \), let \( H \) be some event dependent on the nodes position. Let \( \delta S \) be a small part of \( S \). Crofton’s formula states that:

\[
\delta P[H] = n(P[H|\xi_1 \in \delta S] - P[H])S^{-1}\delta S
\]  

(4)

We can therefore obtain:

\[
P(X > r) = \frac{1}{r} \int_r^{2r} \frac{2r - x}{r^2} dx
\]

\[
P(X > r) = \frac{1}{2}
\]

The total number of paths to be removed is therefore:

\[
N_{\text{rem}} = \frac{r^2 \epsilon^2}{2}
\]  

(5)

Let \( N_F(\alpha_1) \) be the number of paths going through \( F \) at \( P_1 \). \( N_F(\alpha_1) \) is the result of subtracting Eq. 5 to Eq. 3.

The next step of the computation consists in determining the total number of paths for all the points located on \( C_F \). We first perform a transformation of \( P_1 \)’s euclidian coordinates into polar coordinates:

\[
\begin{align*}
x_1 &= r_F \cos \theta_F + \frac{r}{2} \cos \alpha \\
y_1 &= r_F \sin \theta_F + \frac{r}{2} \sin \alpha
\end{align*}
\]

The maximal number of paths \( N_{\text{paths}} \) going through \( F \) can therefore be obtained by summing up all the possible source-destination pairs on the tangents to \( C_F \).

\[
N_{\text{paths}}(r, \theta) = \int_0^{2\pi} N_F(\alpha)d\alpha
\]  

(6)

This can be evaluated with analytical methods.

2) Expected Number of Forwarding Nodes: Let us define the expected progress as the distance covered in 1 hop [7]. This parameter is of significant interest in our computation as it is directly related to the number of hops along a path from a source to a destination. The greater the expected progress, the smaller the number of hops. This parameter depends on the network density and node distribution.

Let \( z \) be the maximum expected progress. With a uniform nodes distribution, the number of nodes follow a Poisson distribution. To have a maximum expected progress \( z \), there should be at least one node in the area located between 0 and \( z \), that is to say the probability \( p_0 \) that there is no node between 0 and \( z \) should be very small:

\[
P(N = 0) = e^{-\rho z} = p_0
\]

We can thus derive the following equation:

\[
\epsilon = \frac{-\log(p_0)}{\rho z}
\]  

(7)

The expected relay traffic per node \( \lambda(r, \theta) \) is therefore:

\[
\lambda(r, \theta) = \frac{\lambda}{2} \frac{1}{\rho z} N_{\text{paths}}(r, \theta)
\]

(8)

D. Validations

To validate our analysis, we used two methods:

1) We implemented a routing algorithm based on node positioning

2) We computed the total relay traffic in a single path routing strategy and compared it against the total relay traffic in a 2-path routing strategy.

1) Iterative Position-based Multipath Routing Algorithm: Given the assumptions of our work (fixed wireless backbone and possibility to easily obtain the nodes positions), we propose a localization-based routing protocol.

The algorithm is implemented as follows. Let \( V \) be the set of nodes, \( S \) the source node, \( T \) the destination node, \( N_c \) the current relay node, \( N \) the next hop node and \( N_{\text{orth}} \) the orthogonal projection of \( N \) on \( (S, T) \). The algorithm consists in iteratively finding the next hop node on each path within the transmission range of the current relay node and satisfying the interference constraints (i.e. the chosen node should be in one of the bands described in Fig. ??).

Algorithm 1 Multipath Routing Algorithm

\[
\begin{align*}
N_c &= S \\
&\text{set} = V \\
&\text{current\_distance} = \text{dist}(S, T) \\
\text{while} \ & \not\text{set} = \emptyset \ do \\
&\text{if} \ \text{dist}(N, N_{\text{orth}}) > (r/2) \ &\& \ \text{dist}(N, N_{\text{orth}}) < (r/2) + \\
&\ &\& \ \epsilon \ &\& \ \text{dist}(N, N_c) < r \ then \\
&\ &\text{if} \ \text{dist}(N, T) < \text{current\_distance} \ then \\
&\ &\ &\ N_c = N \\
&\ &\ &\ \text{current\_distance} = \text{dist}(N, T) \\
&\ &\end{if} \\
&\text{end if} \\
&\text{set} = \{ N \} \\
&\text{pick} \ N \in \text{set} \\
\text{end while}
\]

The algorithm is run for each source-destination pair.
2) Single path routing: In order to determine the number of nodes involved in the total relay traffic for a single path strategy, we need to compute:

1) the total number of paths
2) the probability that a path length is less than the transmission distance (i.e. the source and destination can communicate directly) in order to exclude these paths from the set of feasible paths
3) the average path length \( M(R) \) between two nodes located in a disk of radius \( R \) given that the path length exceeds the transmission range

The total number of paths \( N_{paths} \) is straightforwardly obtained:

\[
N_{paths} = \rho \pi R^2 \ast (\rho \pi R^2 - 1)
\]  
(9)

The probability density for the distance between two random points located in a circle of radius \( R \) can be expressed as [6] [18]:

\[
p(x) = \frac{2x}{R^2} \frac{2}{\pi} \arccos\left(\frac{x}{2R} - \frac{x}{\pi R} \sqrt{1 - \frac{x^2}{4R^2}}\right) \tag{10}
\]

Therefore, the probability that the distance between 2 nodes exceeds the transmission range can be derived easily:

\[
P(x > r) = \int_{r}^{2R} p(x)dx \tag{11}
\]

Finally, we need to calculate the mean distance between two nodes randomly dropped in a disk given that the distance between these two nodes is greater than a distance \( r \) (transmission range). Let \( D \) be the mean distance between a node \( A \) located on the circumference of a circle of radius \( R \) and any other node located in the circle whose distance exceeds the transmission radius. \( D \) can be expressed as follows:

\[
D = \frac{1}{\pi R^2} \int_{r}^{2R} 2x^2 \arccos\left(\frac{x}{2R}\right)dx \tag{12}
\]

\[
D = KR
\]

with \( K = \frac{16}{\pi^3}(-\frac{2}{3}\cos^3(\alpha) + \frac{1}{2}\sin(\alpha) - \frac{1}{6}\sin^3(\alpha)) \) and \( \alpha = \arccos\left(\frac{r}{2R}\right) \).

Therefore, for any two points located in a circle of radius \( R \), the mean distance is:

\[
M(R) = \frac{4KR}{5} \tag{13}
\]

From Eq. 9, Eq. 11 and Eq. 12, we can finally deduce the total relay traffic \( \lambda_{tot} \):

\[
\lambda_{tot} = N_{paths} \ast P(x > r) \ast \left(\frac{M}{r}\right) - 1 \ast \lambda \tag{14}
\]

The implementation of the methods previously described has been realized in Matlab 6.4. Both methods necessary return the same results that is referred as “theory” in Fig. 4. We can observe that both the proposed mathematical formulation and the theoretical formulation match closely.

V. THROUGHPUT ESTIMATION: THE MULTIPLE SOURCE-DESTINATION SCENARIO

In the previous section, we demonstrated that under certain conditions, namely when the paths do not interfere, the network performance in terms of throughput and end-to-end delay can be significantly improved. However, this is without considering the effects of concurrent transmissions between different source-destination pairs. Let us look at the following scenario with two flows from A to B and from C to D (Fig. 5). The nodes are 200m apart, and the same traffic characteristics are assumed for both flows (CBR traffic at 0.005 packets/s and UDP transport protocol).

To estimate the impact of the cross traffic on the nominal capacity, we determine the maximum collision domains for a single path and multipath scenario. The bottleneck links are represented by large solid arrows in Fig. 6). With a single path routing approach, the maximal achievable throughput is \( B/8 \) whereas in a multipath routing strategy, the upper bound is as \( B/7 \ (8* \lambda/2 + 3\lambda) \).
We confirmed these results via simulations implemented in NS-2 (Fig. 7). We can observe that multipath routing still achieves slightly better performance than single path routing. As expected, the throughput for both the single path scenario and the multipath scenario degrades compared to the performance results obtained in Section III due to the increased level of interference. The benefit of multipath routing over single path routing becomes quite insignificant and is expected to vanish if the actual cost of the paths establishment is accounted for.

![Figure 7. Impact of cross traffic on the throughput](image)

VI. CONCLUSION

The need to provide pervasive communication at anytime, anywhere and the necessity to cope with the existing constraints of wireless environments opened new research avenues to improve the network utilization and optimize the network capacity. Multipath routing has been put forward as a potential solution to address these issues, and many proposals have been propounded. However, the problem of interference has not been properly addressed, which questions the efficiency of these protocols in real-world deployments.

In this paper, we focused on two related problems: 1) the estimation of the throughput if only the interference of a single source-destination pair is considered; and 2) the impact of interference when multiple source-destination pairs are considered. We provided an evaluation of the throughput for a 2-path routing scheme while accounting for the interference of concurrent data transmissions for a given source-destination pair. We demonstrated the complexity of the problem and derived an analytical formulation of the expected throughput at each node considering a uniform node distribution. We also proposed a simple method to establish multiple paths between a source node and a destination node based on their coordinates. With multiple source-destinations pairs, although a more efficient network utilization due to a better load balancing can justify the use of a multipath routing strategy compared to single path routing, the benefits of multipath routing in terms of throughput quickly vanish when interference is accounted for.

Therefore, multipath routing can only be of interest if the traffic distribution is known and if the effect of interference can be properly evaluated. However, in most cases, assuming that such information is known or can be obtained easily is unrealistic particularly due to the instability of network conditions in wireless environments and the staleness of information (low convergence due to medium contention). In such context, multipath routing does not appear as a sound routing strategy.

REFERENCES